ZERO-DIMENSIONAL MODELS OF THETA-PINCH AND OCTOPOLE DEVICES

By

R. P. VIJAY

NUCLEAR ENGINEERING AND TECHNOLOGY PROGRAMME
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

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CERTIFICATE

This is to certify that this work entitled "ZERO-DIMENSIONAL MODELS OF THETA-PINCH AND OCTOPOLE DEVICES" has been carried out by Mr.R.P.VIJAY under my supervision and has not been submitted elewhere for a degree.

K. Sri Ram

(K. SRI RAM)
Professor
Nuclear Engineering & Technology Programme
Indian Institute of Technology
KANPUR-208016

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R.P. Vijay
(R.P.VIJAY)

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NOMENCLATURE

V_D = velocity of diffusion

C = speed of light

kT = temperature in eV

$\dot{p}$ = kinetic pressure

T_e = electron temperatures (eV)

T_i = ion-temperature (eV)

n = number of electron or ion-densities

B = magnetic field in gauss

n_o = neutral density

a = plasma radius (cm)

ω_c = plasma frequency

$\eta_\perp$ = resistivity

L = length of plasma column

λ_{ii} = ion-ion mean free-path

U_e = average electron energy density (10^9 eV/cm^3)

U_i = average ion energy density (10^7 eV/cm^3)

A_o = obstacle area (cm^2)

P = microwave power (watts)

f = microwave frequency (GHZ)

Q_o = unperturbed cavity Q_o of plasma chamber

t = time (sec)

τ_{th}^S = thermal conduction time of species

τ_p = plasma column particle confinement time

ε_S = average energy loss per species

τ_{eq} = electron energy equilibration term

CHAPTER-1

The upsurge of our indutrial civilization in the first half of the twentieth century was founded upon fossilfuels- coal and oil.But there are many "have-not" nations and even contries rich in this fuels are now seeing rapid depletion of their reserves At this juncture uranium has come to the rescue as a hope for the future. The worlds uranium and thorium, it is estimated, represent an energy reserves,somewhere between ten and hundred times larger than coal. Even so fissinable fuels too are an exhaustible supply. At the rate at which the world's energy needs are expanding practically all the economically recoverable uranium could as coal, might be exhausted within another century or so.

Besides limited uranium reserves, fission power also presents a more immediate problem; namely, disposal of its radioactive wastes.

All this helps to explain the drive, if not race, to find out whether thermonuclear power can be tapped and put to work. If the fusion reaction can be made to yield

power , it will solve forever both the fuel supply problem and the problem of radioactivewastes. The basic fusion element deuterium, is as inexhaustible as the oceans, and fusion produces no appreciable amount of radioactive by products.

Nuclear fusion is not exactly a new phenomenon . It has been generating the power of the Sun and other stars for billions of years. But to create and control fusion power on earth is a problem of a totally different order from harnessing fission. It is undoubtedly the most difficult project ever presented to scientists and engineers.

The fission releases energy, because part of the mass of the fusing nuclei is transformed into energy according to Einstein's famous equation $E= mc^2$. A continuous fusion reaction, is analogous to the familiar process of combustion. To stick together or fuse, the molecules must collide violently, which means the material must be heated. Three condition are needed to burn a chemical fuel and harness its heat to do work; (1) The fuel must be raised to its ignition point.(2) There must be enough of

it to sustain a continous reaction. (3) The energy released must be tapped in a controled manner. Now precisely the same condition are needed to make a nuclear fusion reaction go and do useful work. The great difference is that for a fusion reaction the ignition point is rather high- hundreds of millions of degrees centigrade.

This one condition - the attainment of which was quite unthinkable on the earth until recently - underlies all our problems. The answer to the question "what is a controlled fusion reactor" is best given today by merely saying that it is a device within which appropriate isotopes of light elements could be caused to undergo nuclear fusion, the end result being the controlled production and extraction of useful quanitities of energy, in excess of that required to operate the device.

Among the nuclear reactions which appear promising for a controlled fusion reaction are those involving the various isotopes of hydrogen, helium and lithium. Typical reaction are given Table (1). But unfortunately nuclear

TABLE 1.1

TYPICAL FUSION REACTIONS

1.	D+D	=	^{3}He +n	+ 3.25	MeV
2.	D+D	=	T +p	+ 4	MeV
3.	D+T	=	^{4}He +n	+ 17.6	MeV
4.	^{3}He+D	=	^{4}He +p	+ 18.3	MeV
5.	^{6}Li+D	=	$2\,{}^{4}_{2}He$	+ 24.4	MeV
6.	^{7}Li+p	=	$2\,{}^{4}_{2}He$	+ 17.3	MeV

fusion reactions have significant reaction cross-section at relative energies ranging between ten keV and hundred keV. At these temperature the matter is completely ionized and exists in the form of plasma in which the ion and electron densities are equal to maintain charge neutrality.

There are basically two approaches to fusion. The first of these is magnetic confinement of plasma. The reacting plasma is too hot to be contained by materialwalls, and it is therefore proposed to use a "magnetic bottle" i.e a magnetic field configuration which curls up the particle orbits in such way as to greatly reduce their escape rate from the reaction volume. Speaking in a very general way,the confined state of plasma is far from equilibrium, and there are many mechanismsleading to the break up, or dispersal, of the confined plasma. Research over many years has lead to an understanding of these escape routes and many have been eliminated. There is , however one phenomenon which is not yet understood; it was discovered by Bohm in 1949,

According to him, there exists an amomalous diffusion process in a magnetic field B. The particles diffuse with a velocity given by

$$V_D = -\frac{ckT}{16cn_c B}\nabla n_e = -\frac{8.64\times10^3 T}{\gamma_B n_e B}\nabla n_e$$

where γ_B is a factor which theory cannot account for in a straightforward way and which according to Bohm is 16. Much research in recent years has consisted of **attempts to** discover which circumstances are reponsible for this anomalous diffusions, presumably caused by turbulent fluctuation process.

Putting, $\nabla n_e = n_e/r$

shows that Bohm diffusion limits the life time, $T = r/v_D$, of a plasma confined in cylindrical geometry with a radius r , to a value approximately given be

$$T = 10^4(\gamma_B \ \gamma_i \ r^2 \ B)/T \qquad (1)$$

where γ_i is an improvement factor which may be raised by controlling the fluctuation level. There B is measured in **Gauss** and T in Kelvin so that for $T = 10^8$ and a field of hundred kG and r = 100 cm, T= 16ms , if $\gamma_i = 1$.

Magnetic confinement systems are characterized by the ratio of thermal energy 2nkT and magnetic energy $B^2/8$,

$$\beta = 16\pi n\, kT/B^2$$

The parameters of a possible reactor can be determined if we also consider the power loading of the reactor walls. A figure of $500W/cm^2$ is commonly taken as a reasonable value. For plasma contained within a radious r the number - N of particles per unit length is given using(1) by $N= r^2 n = 10^4 \; n\, T\tau / B$ and the power W reaching unit area of a wall of radius R is (using (2))

$$W = \frac{3NkT}{2\,RT} = \frac{3x10^{14}\; B\; T}{.32\quad R\,\gamma} \quad , \; \gamma = \gamma_B \gamma_i$$

Again for a temperature of $T= 10^8 K$, we find that the wall radius must be at least

$$R= 5.9\pi\, B\beta/\gamma \qquad (3)$$

Now R must be larger than r and this condition yields,

$$B \geq 3\, (\gamma/E_p \beta^3)^{1/5} \text{ kilo gauss} \qquad (4)$$

For the minimum value of B when r is determined from above.

A reactor with R=100cm is already a very large device. Adopting this value and $B = 2\times 10^2$ kG which is practicable, we see from (3) that must be at least 1800 to satisfy criteria laid in (3) . With this value and E_p =0.01 B must be larger than 180 kG which is consistent with our assumption of 200kG. Such values of have hot been achieved so far, even at temperatures smaller than thermonuclear onees but it may well be possible to reach this goal. We have presented these estimates to show that the search for alternative methods for obtaining fusion power is still necessary.

The foregoing data imply confinement times of 1ms and densities of $10^8/cm^3$ not easy to maintain for sufficiently long times for extracting power.

This has led to an alternative scheme of confinement namely inertial confinement by lasers. It is for this reason that microexplosions involving much higher densities during much smaller times are tried using lasers. We now tern to the

problem of heating and compressing a pellet containing a deuterium-tritum misture. We see it now as a spherical object with a radius of the order of 100 micrometers . The scheme is to illimunate this target as uniformly as possible with intense laser light whereupon some light will be absorbed and materials will evaporate from the pellet surface. At high temperatures it will ionize and form a rapidly expanding plasma mantle. Under intense illumination, the plasma density may rise to such an extent as to make it opaque to laser light. The heat from the absorption region will be conducted to the pellet where it will cause further evaporation. The pressure of the plasma so formed may become very high that a shock wave is launched into the target. The mass loss of the pellet by fast evaporation is called ablation.

Thus accofding to our criteria, with $E_p = 0.01$, $nT = 10^{16}$, an explosion lasting 100ps would correspond to a density of 10^{26} $/cm^3$ which would be just sufficient provided that the fuel could be kept at this density during this time.

Following is a brief review of the various fusion devices being experimented on.

THE PINCH Device

The theory of the constructed gas current was first developed by W.H.Bennet in 1934. A few years later a different approach was presented independently by L.Tonks who used the term "pinch effect" in the sense in which it is employed at the present time. Subsequently, a number of other theoretical analyses have been published of the construction of a current filament due to the action of the azimuthal magnetic field generated by the current itself. Using the Bennet relation $I^2 = 2N_e k\ (T_e+T_i)c^2$ it appears that the attainment of a sufficiently high temperature in a pinch discharge depends on the ablity to pass current of that magnitude.

Since the temperature in the quasi-equalibrium pinch varies as the square of the current. But equilibrium temperature is limited as the rate of energy loss will increase rapidly with the pinched discharge current.

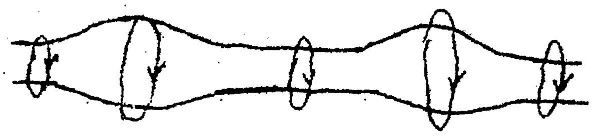


FIGURE 1.1 SAUSAGE INSTABILITY

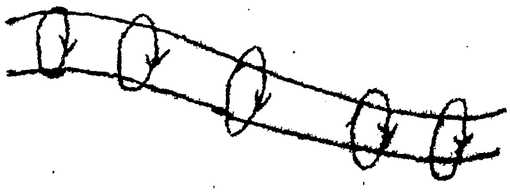


FIGURE 1.2 KINK INSTABILITY

A considerable amount of work has been done with "fast discharges" in which the constricted plasma is established in a time that is short in comparaision with the sound transit and the magnetic field diffusion times. The dynamic pinch (or shock pinch) as described above has received a great deal of attention experimentally since it offers, in principle, a way to obtain high average particle energy in a plasma. When the pinch reaches its smallest radius the kinetic energy of the particles in the plasma reach their maximum inward velocity. If the pinched discharge column can be kept contracted long enough to permit randomization, i.e., thermalization of the ion velocity to take place ,a high temperature will result.

One of the major probems associated with the constricted discharge and infact, with any system of charged particles confined by a magnetic field- is that of instability. There are two main types of instablity which leads to the destruction of a pinched plasma. The first often called the sausage type of instability, results from an azimuthally symmetrical localized constrction (and expansion) of the plasma column.

This is diagramatically represented in figure 1.1 Consequently, the constrictions tend to become more and more constricted , whereas the bulges will grow larger and larger until the pinched discharge is completely disrupted.

The second type called kink instability originates with the formation of a band or a kink in the plasma column while the latter retains its uniform , circular cross-section.

This is diagramatically represented in figure 1.2 It is seen that the lines of force of the azimuthal self-magneticfield, due to the current in the plasma, are brought closer together on the inside, but they are further apart on the out side of a bend. Thus once a slight kink developes ,it will grow in size until the pinched discharge strikes the walls and is cooled. The plasma will then become diffuse, and fills the containing tube.

A part from the problem of instabilities, the theta pinch devices are also bogged by problems of end losses. This has led to the the development of toroidal devices.

The adiabatic in variance of the magnetic moment in a space dependent mganetic field leads to a magnetic field configuration known as a magnetic mirrors.

The other concept led to development of a toroidal pinch device initially. The most promising of all thermonuclear reactors to date is of the Tokamak series developed in the Soviet Union. This is a toroidal pinch with a very strong toroidal magnetic field B_{ϕ} generated by the current I_{ϕ} A poloidal field results from a large current induced in the plasma by an iron core. The stellarator was initially conceived as a toroidal like steady state confinement. The simplest form is that of a system of coils which generates a magnetic field in a toroidal region of space. By contradistinction with the toroidal pinch the magnetic line of force here are "parallels" of the toroida.instead of mediams. In addition there are a number of other devices like the spheromak and base-ball machine.

Recently , it has been found that in order to achieve a practical fusion device one has to use a high beta fusion device. Given the low-beta of tokamaks, there is renewed interest in theta pinch devices and more recently the REVERED -FIELD PINCH (RFP) .

The main plasma parameter of a thermonuclear configuration are determined by physical and economical consideration. For the output in Thermonuclear energy from fusion reactor to exceed the energy needed to heat the plasma the product of the plasma density and the time T should satisfy the Lawson ceriteria; $nT = 10^{14}2\ /cm^{3}$ where T is the mean time a charged particle can be held in the plasma volume. The mean power density desired from a fusion reacter is determined by the condition that the capital cost of the reactor should be comparable with that of the conventional power sources. The maximum power density possible is limited by the capability of the srounding material to withstand the energy released, without being destroyed. Under these two condition, the power density released by the fusion reacter should be btween 1 and $100W/cm^{3}$. It is clear that power release will be a function of the temperature and the density of the confined plasma. Under the circumstances, it is possible to consider three regions of confinement: these are:

(1) high density low temperature, short life time plasmas.

(2) low density, low temperature, long life time plasmas.

(3) low density, high temperature, long life time plasmas.

Whatever is the form of the magnetic confinement the plasma can always escape from the configuration. Energy losses from a controlled -thermonuclear reaction systems may be divided into two broad categories- direct losses and indirect losses. Among the direct losses are those due to plasma instabilities, radiation, charge exchange, thermal conduction and diffusion. These are all energy losses that take place directly from the plasma itself. The indirect losses are those arising from external circumstances, such as the losses resulting from joule heating in the magnetic field coils and from the conversion of the thermonuclear reaction energy into useful power.

Some of the parameters of a thermonuclear reacting system, notably the dimensions of the reacting chamber and the strength of the confining magnetic field, can be varied in such a manner as to increase the power output and

minimise losses. The broad rules which are applicable are refered to as scaling laws. .

One of the main uses of a zero dimensional simulation program such as this might be to predict scaling laws for magnetic confinement devices.

1.2 SOME USEFUL DEFINTIONS:

In this section, we first give definitions of system parameters like beta, neutral pressure , crowbarring, elastic and inelastic collusion, ion an mean free path,and ambipolar diffusion.

BETA:- It is often convenient to determine the ratio of the kinetic pressure of the particle to the external magnetic pressure. The dimensionless quantity is then defined by

$$\beta = \frac{p}{B_o^2/8\pi}$$

The energy balance equation is given by

$$p + \frac{B^2}{8\pi} = \frac{B_o^{\,2}}{8\pi}$$

This may be written as $\beta = 1 - \frac{B^2}{B_o^{\,2}}$

Since the minimum value of B is zero, the ratio has a maximum value of unity; this would represent the ideal case of a perfectly diamagnetic plasma from which the magnetic field was completely excluded. In this event we have

$$p_{max} = \frac{B_o^2}{8\pi}$$

where p_o is the maximum kinetic pressure of a plasma that can be confined, in a study state, by a external magnetic field of strength B_o.

NEUTRAL PRESSURE:- This is the pressure exerted by the neutral gas in the system. The magnetic region in which the injected ion are trapped contain a large number of neutral gas atoms, owing to the finite initial pressure the system must have.

CROWBARRING :- This is a technique by which the external magnetic field applied to the plasma is forcefully brought to zero after a predetermined amount of time.

ION-ION MEAN FREE PATH:- This is the mean distance travelled by the ion between collisions with oter ion.

PLASMA CONFINEMENT TIME: This is the average time the plasma stays confined before it escape from the magnetic field. This is also equal to 1/e times the plasma decay time.

AMBIPOLAR DIFFUSION: This is a diffusion process which takes place due to the movement of electron and ions across the magnetic field with equal velocities.

1.3 LITERATURE REVIEW:

Computer simulation of various fusion devices using time dependent zero dimensional computer models have been carried out by several authors (8,9,10).

In reference (4) the author has indicated that the model that he has developed was a fast, simple, time dependent, zero dimensional computer model for magnetically confined plasmas. According to the author, this method could used to model other devices or heating methods very easily. According to the author most of the terms do not depend on the particular geometry, containing at most the total plasma volume, and so could be used unchanged in modelling other devices.

Several theoretical studies which describe and losses from theta pinch plasma column have been reported (9,10,8) In the zero dimensional code of Green et al (8) the electron and ion temperatures were assumed equal and particle end losses was included resulting in a solution for temperature which was seperated in spatial&time varialbles. The zero dimensional model developed by Klevans et al includes both uniform plasma profiles as well as the effects of ion thermal conduction and finite thermal conductivity.

In the present thesis work, attempt has been made to include the geometrical theta pinch device as indicated in reference (2) into the model developed in reference(1) to produce a zero-dimensional model of a pinch device where the preionzation has been done using a plasma gun.

1.4 OUT LINE OF THE PRESENT WORK:-

In chapter 2 a zero dimensional model is developed by defining the various energy loss terms. These energy loss terms have been space averaged in order to make them space independent. From this a set of equations, one each for plasma density electron energy density and ion energy

density have been defined. In addition in chapter 2 the energy terms of a theta pinch device have been derived. In chapter 3 we established the method by which set of equations are solved. We also determine in chapter 4 by parameter adjustment, the optimal neutral pressure for obtaining the appropriate variation of temperature. As will be seen latter , any one of the following three situations can occur in a pinch device:

(1) Progressively decreasing electron temperature variation whereas a progressively increasing densities.

(II) Progressively incereasing electron temperature variation whereas a progressively decreasing densities.

(III) Gradually approached peaks of electron ion temperaurs and densities.

These cases are obtained by proper adjustment of neutral pressures. Next the electron and ion temperature variation in case of adiabatic compression studied.

Finally, the summary and the main conclusion of this work are presented in chapter 5.

CHAPTER 2

The computer program (SIMULT) calculates the time dependent, spitially averaged density, electron temperature and ion temperature in a cylindrical multipole device [1]. The program is zero- dimensional in the sense that it predicts only spatially averaged quantities. It has been used mostly to study continuous wave microwave heated plasmas, but pulsed microwave and gun-injected plasmas can also be studied.

The numerical method consists of solving three coupled, time dependent, non-linear, first order, ordinary differential equations, one for the p article density, one for the electron energy density and one for the ion energy density.

$$\frac{dn}{dt} = \underset{(A)}{\frac{\partial n}{\partial t}\text{ ionization}} - \underset{(B)}{\frac{\partial n}{\partial t}\text{ diffusion}} - \underset{(C)}{\frac{\partial n}{\partial t}\text{ obstacles}} - \underset{(D)}{\frac{\partial n}{\partial t}\text{ field decay.}} \tag{1}$$

$$\frac{dU_e}{dt} = \underset{(E)}{\frac{\partial U_e}{\partial t}\text{ microwaves}} - \underset{(F)}{\frac{\partial U_e}{\partial t}\text{ excitation}} - \underset{(G)}{\frac{\partial U_e}{\partial t}\text{ ion collision}} - \underset{(H)}{\frac{\partial U_e}{\partial t}\text{ bremsstrahlung}} - \underset{(I)}{\frac{\partial U_e}{\partial t}\text{ synchroton}} - \underset{(J)}{\frac{\partial U_e}{\partial t}\text{ thermal conduction}} \tag{2}$$

$$\frac{dU_i}{dt} = \underset{(K)}{\frac{\partial U_i}{\partial t}\text{ electron collisions}} - \underset{(L)}{\frac{\partial U_i}{\partial t}\text{ charge exchange}} - \underset{(M)}{\frac{\partial U_i}{\partial t}\text{ thermal conduction}} \tag{3}$$

The first equation (1) describes the electron and ion densities in terms of ionization, diffusion, obstacle losses and field decay. The second equation (2) describes the electron energy density in terms of microwave heating, excitation energy, electron-ion collision, bremsstrahlung losses, synchrotron losses and thermal conduction losses. The third equation (3) describes the ion energy density in terms of ion-electron collisions, charge exchange and thermal conduction losses.

The computer results could not be compared with experimental values because all the required parameters in the pinch device were not available.

The three equations are solved by the simplest method of successive iteration using the current value of each parameter. The electrons and ion-temperatures are incremented according to :

$$\Delta T_e = (\Delta U_e - T_e \Delta n)/n$$

$$\Delta T_i = (\Delta U_i - T_i \Delta n)/n$$

Δn is change in n in Δt.

The iteration step Δt can be adjusted to insure any desired degree of accuracy. The initial conditions (n, T_e, T_i) can be set arbitrarily.

In this study a time dependent magnetic field as specified below.

$$B = B_o e^{-t/\tau} \sin\omega t \qquad \omega = 200\ /\mathrm{sec} \quad \tau = 10^{-2}\mathrm{sec.}$$

In this study microwave power pulse as specified in [1] was used.

The neutral hydrogen gas is assumed to consist of thermal $(T = T_W)$ molecules. The initial value of the neutral density is an experimental parameter which must be specified.

Reference [1] indicates that the neutral (hydrogen) density is calculated at each time step from the pressure (p) as follows:

$$n_o = 322p \exp[-3.07\times10^{-4} \frac{na}{T_e^{1/8}} e^{-17/T_e}]$$

$$+ 12.88p \exp[-1.7\times10^{-5} \frac{na}{T_e^{1/8}} e^{-17/T_e}].$$

The first term represents the thermal (0.025eV) neutrals and n_{FC} represents Franck Condon (7eV) neutrals. The exponential factors account for the finite penetration depth of neutrals into the plasma. The thermal neutrals are assumed to be at wall-temperature (T_W). The experiment is assumed to take place on a time scale short compared with the pumping time so that no neutrals are lost to the vacuum pumps. When the neutral density is increasing in time, the rate of increase of each component is limited to:

$$\frac{dn_{TH}}{dt} \geqslant \frac{1.9\times10^{-5}}{a} n_{TH}$$

$$\frac{dn_{FC}}{dt} \geqslant \frac{3.4\times10^{-6}}{a} n_{FC}$$

representing the transit time of a neutral from the edge to the center of the plasma.

(A) IONIZATION-The ionization rate for a Maxwellian electron distribution in a cold neutral gas, can be written in a form given by Drawn [ref(1)]. For hydrogen the electron neutral

particles migrate across B by a random walk process. When an electron collides with a neutral atom, the electron leaves the collision in a different direction but its phase is changed discontinously.

In such a case, $D_\perp = \dfrac{D}{1+\omega_c^2\tau^2}$

When $\omega_c^2\tau^2 >> 1$, $D_\perp = \dfrac{KT}{m\nu} \times \dfrac{1}{\omega_c^2\tau^2} = \dfrac{KT\nu}{m\omega_c^2}$

$$\omega_c = \frac{eB}{m}$$

$$\Rightarrow D = \frac{KTxn_o x\bar{\sigma}\bar{v} \times m^2}{mxe^2xB^2} = \frac{KTxn_o \bar{\sigma}\bar{v} \, xm}{B^2xe^2}$$

$$T_j = -D_\perp \frac{n}{n} \times n$$

$$= \frac{-KTxn_o x\bar{\sigma}\bar{v}xm}{B^2xe^2} \times \frac{\partial n}{\partial x}$$

$$= \frac{Kxnn_o T_e}{B^2xa^2}$$

Consider the I term :

In a fully ionized plasma, $D_\perp = \dfrac{\eta_\perp xnx\Sigma kT}{B^2} = \dfrac{\eta nT}{B^2}$ [T is in eV]

Electron current $\tau = \dfrac{\eta_\perp \, xn \quad n}{B^2} \times T$

$$\frac{n}{\quad} = \frac{Kx\eta_\perp \, xnx \quad n}{B^2}$$

Since $\eta_\perp$ = constant times $(kT)^{-3/2}$ [Refrence 4]

collisional ionization rate is $\frac{dn}{dt}$ ionization $= \frac{-371e^{-S}}{1+S\ T_e}(\frac{1}{20+S}$ $+ \ln(1.25(1+1/S)))$.

where $S = 15.6/T_e$.

From another approach the ionization rate for a Maxwellian electron distribution is approximated by an anlytical fit for the data given in [11]. In view of the inavailibility of ref[11] a analytic fit of the cross-section for ref[12] was performed and a similar variation was found. In general the ionization is largely proportional to the electron temperatures.

$$\frac{dn}{dt}\text{ ionization} = 50.4\frac{nn_o}{T_e^{1/8}}\ e^{-17T_e}$$

(B) DIFFUSION- Classical diffusion due to electron-ion and electron neutral collisions is given by

$$\frac{dn}{dt}\text{ diffusion} = \underset{(I)}{-0.33\frac{n^2}{B^2a^2T_e^{1/2}}}\ \underset{(II)}{-10^{-3}\ \frac{nn_oT_e}{B^2a^2}}$$

The above term has been satisfactorily derived as follows.

Consider the II term. In a magnetic field charged particles will move along B as

$$\tau_j = nV_j = \pm\mu_j nE - D_j\nabla n.$$

If there were no collisions, particles would not diffuse at all in the perpendicular direction. When there are collisions

$$\frac{\partial n}{dt}\,\text{diff} = \frac{\text{constant x } n x n_o x T^{-3/2} x T^{1/2}}{B^2 x a^2}$$

$$= \frac{\text{constant } x n x n_o}{B^2 x a^2 x T^{1/2}}$$

Here ambipolar electric field has been considered.

(C) OBSTACLES: Hoop supports and probes which intercept the plasma are assumed to collect particles at a rate given by the ion saturation current for a isotropic plasma.

Current density J = nx velocity

$= \text{constantx} \sqrt{(T_e+T_i)}$

A_o/a^2 is a factor which determines the fraction of the area occupied by the obstacles.

Hence we have $$\frac{\partial n}{\partial t}\,\text{obstacles} = \frac{2.0\times10^5\, n A_o \sqrt{(T_e+T_i)}}{a^2 L}$$

(D) FIELD DECAY : When $\frac{dB}{dt}$ is negative field lines are leaving the machine, and plasma is assumed to leave also at a rate given [1]

$$\frac{\partial n}{\partial t}\Big|_{\text{field decay}} = -\min[0, \frac{n}{B}.\dot{B}]$$

(E) MICROWAVE HEATING : Ref[1] calculate the microwave heating rate as

$$\frac{\partial T_e}{\partial t}\Big|_{\text{microwaves}} = \frac{2\times10^9 P}{(1+n_c/n)\,a^2 L}$$

where n_c is the density above which the microwaves are totally absorbed. In general, according to ref [1] n_c is given very roughly by $n_c = 487f^2/Q_o$.

Here is a brief discussion about the same. The incidence of a microwave on a plasma gives rise to the following phenomena (1) Absorption (2) Diffusion, reflection and secondary emission (3) Phase lag, caused by differences in refractive indices. (4) Modification of the polarization and birefringence among other phenomena.

Here Q factor is the measure of the sharpness of resonance.

Instead of a quantitative derivation of the expression on the R.H.S., a dimensional analysis in order to understand the derivation of the various terms is given below.

Consider R.H.S. We have $\frac{\text{Watts}}{\text{cm}^3\text{x(factor)}} = \frac{\text{energy}}{\text{timexarea}}$ = L.H.S.

A little more meaning could be given if one considers the process to be due to transit time magnetic pumping. If it is assumed that there is equipartition of energy between ions and electrons, $P = 3nk\frac{dT}{dt}$; This assumes that the phase velocity ω/k for optimal heating is of the order of the mean ionthermal velocity.

$$\omega^2{}_k2 = \left(\frac{2kT_i}{M}\right) \qquad P = 3nk\frac{dT}{dt} \qquad P = K\frac{dU}{dt}$$

Here $K = \frac{(1+n_c/n)a^2L}{2\text{x}10^9}$.

(F) EXCITATION : An electron can also experience a large number of different types of inelastic collinsions with atom, ions and molecules. In a collision with one of these heavy particles, an electron may merely excite the heavy particle. The cross-section will be dependent on the energy of relative motion and the particular quantum level excited. Reference[1] has assumed that there is a 30eV/collision. Hence we have

$$\frac{\partial U_e}{\partial t}\text{ excitation} = -30\,\frac{dn}{dt}\text{ ionization}\ .$$

(G) ION COLLISIONS : Consider an encounter between two charged particles. Here each particle moves in a hyperbola relative to the CM of the two particles. The distance of closest approach is p ''the impact parameter''. Consider a test particle with mass m, a charge Ze/c and a velocity $\underline{W}$. Then $\Delta\underline{W}$ represents the charge of velocity of the test particle.

Consider the Maxwell-Boltzmann distribution

$$f^{(o)}(W) = \frac{nL^3}{3/2}\, e^{-L^2W^2}$$

where n and m are the particle density and the mass of the particle in question, and L is defined as $L^2 = \frac{m}{2kT}$.

To measure the rate of diffusion in the W_X direction, we set N equal to the average number of encounters in one second. The resultant value of $(\Delta\bar{W}_X)^2$, measuring the increase of velocity dispersion of a group of particles/second

may then be denoted by $<(\Delta W_X)^2>$. This quantity is known as ''diffusion coefficient''.

When the distribution function of the field particles is assumed to obey a Maxwell-Boltzmann distribution only three independent diffusions need to be considered, $<\Delta W_{11}>$, $<(\Delta W_{11})^2>$, and $<(\Delta W)_{\perp}^2>$. The first quantity represents the rate at which moving test particles are slowed down by interaction with the field particles. The quantity $<(\Delta W_{11})^2>$ represent the rate of increase of $(\Delta W)^2$ in a direction parallel to the origional motion of the particles. The corresponding quantity $<(\Delta W)_{\perp}^2>$ represents the rate of increase in the perpendicular direction

$= \overline{h/p_o}$ where h is the Debye Shielding distance and p_o the impact parameter

$$= \frac{3}{2ZZ_1e^3}\left(\frac{k^3T^3}{\pi n_e}\right)^{1/2}$$

As $\ln\Lambda$ increases T increases; $\ln\Lambda$ decreases as n increases.

Detailed computation of the diffusion coefficients have been carried out by Chandrasekhar [3]. The resultant formulas for the three diffusion coefficients are

$$<\Delta W_{11}> = -A_D L_1^2\left(1+\frac{m}{m_1}\right)G(L,W)$$

$$<(\Delta W_{11})^2> = \frac{A_D}{W}G(L,W)$$

$$<(\Delta W)^2> = \frac{A_D}{W}\quad(L,W)-G(L,W)$$

where the ''diffusion constant'' A_D is defined by

$$A_D = \frac{8\pi e^4 n_1 Z^2 Z_1^2 \ln}{m^2}$$

$\varphi(x)$ is the usual error function,

$$\varphi(x) = \frac{2}{\pi^{1/2}} \int_0^x e^{-y^2} dy$$

and the function $G(x)$ is defined in terms of $\phi(x)$ by the relationship

$$G(x) = \frac{\varphi(x) - x\varphi'(x)}{2x^2}$$

The ''time of relaxation'' denotes the time in which collisions produce a large alteration in some original velocity distribution. If one considers certain times which are defined in terms of the diffusion coefficients, we have 't_D' , 't_D' is the time between collision in which reflection gradually deflect the test particles by 90^o. In other words

$$<(\Delta W\ )^2>\ t_D = W^2.$$

An energy exchange term t_E may also be defined by the relation

$$<(\Delta E)^2>\ t_E = E^2$$

The change of energy, ΔE in a single encounter is given by

$$\Delta E = \frac{m}{2}\{2W\Delta W_1 + (\Delta W_{11})^2 + (\Delta W\ )^2\}$$

Spitzer has determined the rate at which equipartition of energy is established between two groups of particles having

The actual calculation is difficult because $\bar{V}$ and $\dot{\bar{V}}$ are not well known. Hence we assume that on electron of speed v has an impact parameter b with respect to an ion of charge q_i which is assumed to be infinitely massive. The electron acceleration is

$$\dot{v} \approx \frac{q_e q_i}{4\pi \varepsilon_o m b^2}$$

with $v^2 << C^2$ and neglecting the cross-product, we have

$$\frac{dU}{dt} = \frac{q_e^2 \dot{v}^2}{6\pi \varepsilon_o C^3}$$

$$\frac{dU}{dt} = \frac{q_i^2 q_e^4}{96\pi^3 \varepsilon_o^3 C^3 m_e^2 b^4}$$

The total energy loss rate by the electron colliding with ions at all impact parameters is

$$\frac{dU}{dt} = \frac{q_i^2 q_e^4 (n_i v_e)}{96\pi^3 \varepsilon_o^3 C^3 m_e^2} \int_{b_{min}}^{b_{max}} \frac{db}{b^4}(2\pi b)\, \frac{b}{v_e}$$

Taking $b_{max} = \infty$ and $b_{min} = \dfrac{h}{2\pi m_e v_e}$

$$W_x = \frac{q_i^2 q_e^2 n_i n_e}{24\pi \varepsilon_o^3 C^3 m_e h} \sqrt{\frac{8kT_e}{m_e}}$$

$$= 48\text{x}10^{-37}\ Z^2 n_i n_e T_e^{1/2} \quad \text{Watt/cm}^3$$

Maxwellian velocity distributions, but different kinetic temperature. T and T_1. Next he used (A) and averages this over a Maxwellian velocity distribution.We have

$$\frac{dT}{dt} = \frac{T_1 - T}{t_{eq}}$$

where t_{eq} is the time of equipartition given by

$$t_{eq} = \frac{3mm_1 k^{3/2}}{8(2\pi)^{1/2} n_1 Z^2 Z_1 e^4 \ln}\left(\frac{T}{m} + \frac{T_1}{m_1}\right)^{3/2}$$

This implies $\frac{dT}{dt} = \frac{(T_e - T_i) \times 8(2\pi)^{1/2} n_1 Z^2 Z_1^2 e^4 \ln}{3mm_1 k^{3/2}\left(\frac{T}{m} + \frac{T_e}{m_1}\right)^{3/2}}$

Now $\frac{dU_e}{dt} = 3n\,\frac{dT}{dt}$

$$\frac{dU_e}{dt} = \frac{Kxn^2 x(T_e - T_i)}{\left(\frac{T}{m} + \frac{T_e}{m_1}\right)^{3/2}}$$

which compares well with $\frac{dU_e}{dt} = \frac{6\ln^2 (T_e - T_i)}{T_e^{2/3}}$.

(H) <u>BREMSSTRAHLUNG</u> : According to classical electromagnetic theory, the total power U radiated from a charge q moving with a velocity $\bar{V}$ and acceleration $\dot{v}$ is

$$\frac{d\bar{U}}{dt} = \frac{q^2}{6\pi\,\varepsilon_o C^3}\,\frac{\dot{v}^2 - (\bar{V} x \dot{\bar{V}})^2/C^2}{(1 - v^2/C^2)^3}$$

(I) <u>SYNCHROTRON RADIATION</u> : Again according to basic electromagnetic theory,

$$\frac{dU}{dt} = \frac{q^2}{6\pi \varepsilon_o C^3} \frac{\dot{v}^2 - (\bar{v}\times\dot{\bar{v}})^2/C^2}{(1-v^2/C^2)}$$

In this case $(v/C)^2 << 1$ and $\dot{v} = \dot{v}_{\perp}$.

The electrons are slightly relativistic. Their angular frequency of rotation ω_o is determined by their relativistic mass, so that

$$\omega_o = \frac{q_e B[1-(v^2/C^2)]^{1/2}}{m} = |\omega_b| (1-v^2/C^2)^{1/2}$$

Substiuting $\dot{v}_{\perp} = \omega_o v$ we get

$$\frac{dU}{dt} = \frac{q_e^4 v^2 B^2}{6\pi \varepsilon_o m^2 C^3 [1-(v^2/C^2)]}$$

Expressed in terms of momentum,

$$\frac{dU}{dt} = \frac{q_e^4 B^2}{6\pi \varepsilon_o m^4 C^3},$$

which is to be integrated over the relativistic distribution $f(p)$. Since $f(p)$ is isotropic and since these are two direction associated with p but only one associated with p_{11}, we have

$$p_{\perp}^2 = 2\bar{p}^2/3$$

The exact answer appears in terms of the modified Bessel's function $K_3(Z)$ of the third order. The integral definition of the Hankel functions $H_2^{(1)}(Z)$ of first kind and second order and the derviative relations for these functions are used to evaluate these integrals.

$$W_c = \frac{e^4 B^2}{3\pi \varepsilon_o m^2 c}\left(\frac{n_c kT_e}{mc^2}\right) \frac{K_3(mc^2/kT_e)}{K_2(mc^2/kT_e)}$$

From the asymptotic expansion of the modified Bessel's function,

$$W_c = \frac{e^2 \omega b^2}{3\pi \varepsilon_o c}\left(\frac{n_c kT_e}{mc^2}\right)\left(1+ \frac{5}{2}\frac{kT_e}{mc^2}\right)$$

$$= 6.2\times10^{-17} B^2 n_e T_e [1+(T_e/204)] \text{ Watts/cm}^2$$

Here ω_b is expressed in terms of B.

(J) <u>THERMAL CONDUCTION</u> : Heat conduction due to classical collisions and to obstacle losses [7] is given by

$$\frac{dU_e}{\partial t}\bigg|_{\text{thermal conduction}} = 2.5T_e \frac{\partial n}{\partial t}\bigg|_{\text{diffusion}}$$

$$+ 5.54T_e \frac{\partial n}{\partial t}\bigg|_{\text{obstacle}}$$

(K) <u>ELECTRON COLLISIONS</u> : Ions are heated by classical collisions with electrons.

$$\frac{dU_i}{dt}\bigg|_{\text{electron collisions}} = - \frac{dU_e}{dt}\bigg|_{\text{ion collisions}}$$

(L) <u>CHARGE EXCHANGE</u> : Reference [1] determined an approximate analytic form for change exchange losses, obtained by averaging cross-section found in ref. [12]

$$\frac{\partial U_i}{\partial t} = \frac{0.0186 n n_o T_i^3}{T_i + 100}$$

Our first study had been to determine the exact expression itself. Later a sixth order polynomial was fitted for the data in ref [12] and found similar variation in the cross section.

(M) <u>ION THERMAL CONDUCTION</u> : Heat conduction due to collisions and to obstacle losses [7] is given by

$$\frac{\partial U_i}{\partial t}\Big|_{\text{thermal conduction}} = 2.5 T_i \frac{\partial n}{\partial t}\Big|_{\text{diffusion}} + 2 T_i \frac{\partial n}{\partial t}\Big|_{\text{obstacles}}$$

~~The first equation (1) describes the electron and ion densities in terms of ionization, diffusion, obstacle losses and field decay. The second equation (2) describes the electron energy density in terms of microwave heating, excitation energy, electron-ion collision, bremsstrahlung losses, synchrotron losses and thermal conduction losses. The third equation (3) describes the ion energy density in terms of ion-electron collisions, charge exchange and thermal conduction losses.~~

The above formulation of ref. [1] was modified in 2.2

to include plasma contraction terms as proposed by ref.[2]. The following terms (N) and (P) contribute to the electron energy density term. whereas the term (O) contributes to the ion energy density term.

2.2 ADDITIONAL TERMS FOR A THETA - PINCH :

(N) A certain amount of work is done on the plasma by the system when the plasma contracts contributing to the increase in electron and ion temperatures. The following derivation is done for the term $P_s \frac{dV}{dt} = nT_s L(\frac{dA_p}{dt})$.

For the theta-pinch device described in Chapter 1 we have the radial pressure balance as

$$B_e^2(r) = B_i^2(r) + 4.03\times10^{-20} n(r)(T_e+T_i)$$

Here we have assumed that the radial inertial effects can be neglected and that the magnetic field has a small curvature. Let $\zeta_a \frac{B_a}{B_e}$.

Assuming a uniform magnetic field profile with

$$B(r) = \begin{cases} B_a & r \leq a \\ B_e\,, & r > a \end{cases}$$

$$n(r) = \begin{cases} 2.48\times10^{19}[B_e^2/(T_e+T_i)](1+\zeta_i^2) & r \leq a \\ 0 & r > a \end{cases} \qquad (1)$$

According to ref. [2] $\bar{n} = n(r)$ for a uniform profile.

$$\bar{n} = 2.48\times10^{19}\frac{B_e^2}{(T_e+T_i)}(1-\zeta_a^2) \quad -2$$

Using the expression $\frac{\partial\varphi}{\partial} = 2\pi a \frac{\eta}{\mu_o} \frac{\partial B_2}{\partial r}\Big|_{r=a}$

where $\varphi = \int_{A_p} B_i(r)\,d\eta.$

Combining equation [ ] and [2], the desired expression for $\frac{d\zeta_a}{dt}$ is obtained

$$\frac{d\zeta_a}{dt} = \zeta_a\left(\frac{1}{A_p}\frac{dA_p}{dt} + \frac{1}{B_e}\frac{dB_e}{dt}\right)$$

From another approach it can be established that

$$\frac{d\zeta_a}{dt} = \frac{1-\zeta_a^2}{2\zeta_a}\alpha \text{ where } \alpha = \frac{2}{B_e}\frac{dB_e}{dt} + \frac{5}{3}\frac{1}{A_p}\frac{dA_p}{dt} + \frac{1}{T_e+T_i}\left[T_e\left(\frac{I}{\tau_{th}^2}\right)\right.$$

$$\left. + T_i\left(\frac{I}{\tau_{th}^i}\right)h(0.25L-\lambda_{ii}) + \frac{2}{3}\frac{1}{\tau_p}(\varepsilon_e+\varepsilon_i)\right]$$

From the above two equations, for a uniform profile, we have

$$\frac{1}{A_p}\frac{dA_p}{dt} = -\frac{3}{(5+\zeta_a^2)}\left(\frac{2}{B_e}\frac{dB_e}{dt} + \frac{(1-\zeta_a^2)}{T_e+T_i}\Delta\right)$$

$$\text{where } \Delta = \frac{2}{3}\frac{1}{\tau_p}(\varepsilon_e+\varepsilon_i)+T_e\left(\frac{1}{\tau_{th}^e}\right)+T_i\left(\frac{I}{\tau_{th}^i}\right)h(0.25L-\lambda_{ii})$$

(O) A terms similar to (N) above is used in the ion energy density expression.

(P) OHMIC HEATING :

The ohmic heating term $\hat{H}_{\Omega} = L \int_{A_p} \eta \; j^2 \; (r) dA$.

With the assumption that the plasma is fully ionized the plasma resistivity

$$\eta_{\perp} = 3.27\times10^{-9} \ln \; / T_e^{3/2} \text{[Ref(4)]}$$

$$j \; (r) \text{ from Ampere's law is } \left| \frac{1}{\mu_o} \frac{\partial B(r)}{\partial r} \right.$$

For a uniform distribution $\frac{\partial B(r)}{\partial r} = \frac{\partial}{\partial r} \left[B_e (0.5+0.5 (\frac{r}{a})^2) \right]$

$$= \frac{0.5 B_e x2r}{a^2} = \frac{B_e xr}{a^2}$$

$$\hat{H}_e = L \int_o^{r_p} 3.27\times10^{-9} x2.2/T_e^{3/2} \; x \; \frac{B_e^2 xr^2}{a^4} \; x \; 2\pi \; x \; r \; x \; dr$$

Here $\quad r_p = \sqrt{A_p/2\pi}$.

CHAPTER-3

PROGRAM DESCRIPTION

3.1 The numerical method consists of solving three coupled, time dependent, non-linear, first order, ordinary differential equations, one for the particle density, one for the electron energy density and one for the ion energy density.

The three equation are solved by the simplest method of succesive iteration using the current value of each parameter. The electron and ion temperatures are incremented according to:

$$T_e = (\Delta U_e - T_e \Delta n)/n$$

$$T_i = (\Delta U_i - T_i \Delta n)/n$$

The iteration step t can be adjusted insure any desired accuracey. The inital condition (n, T_e, T_i) are given as input and can be set arbitrarly.

3.2 The calculation is done in the following manner. Intially we specify the peak microwave PO in wats , IIMAX the number of iterations, DEA the initial neutral density,

TABLE 3.1

LIST OF DEVICES SIMULATED

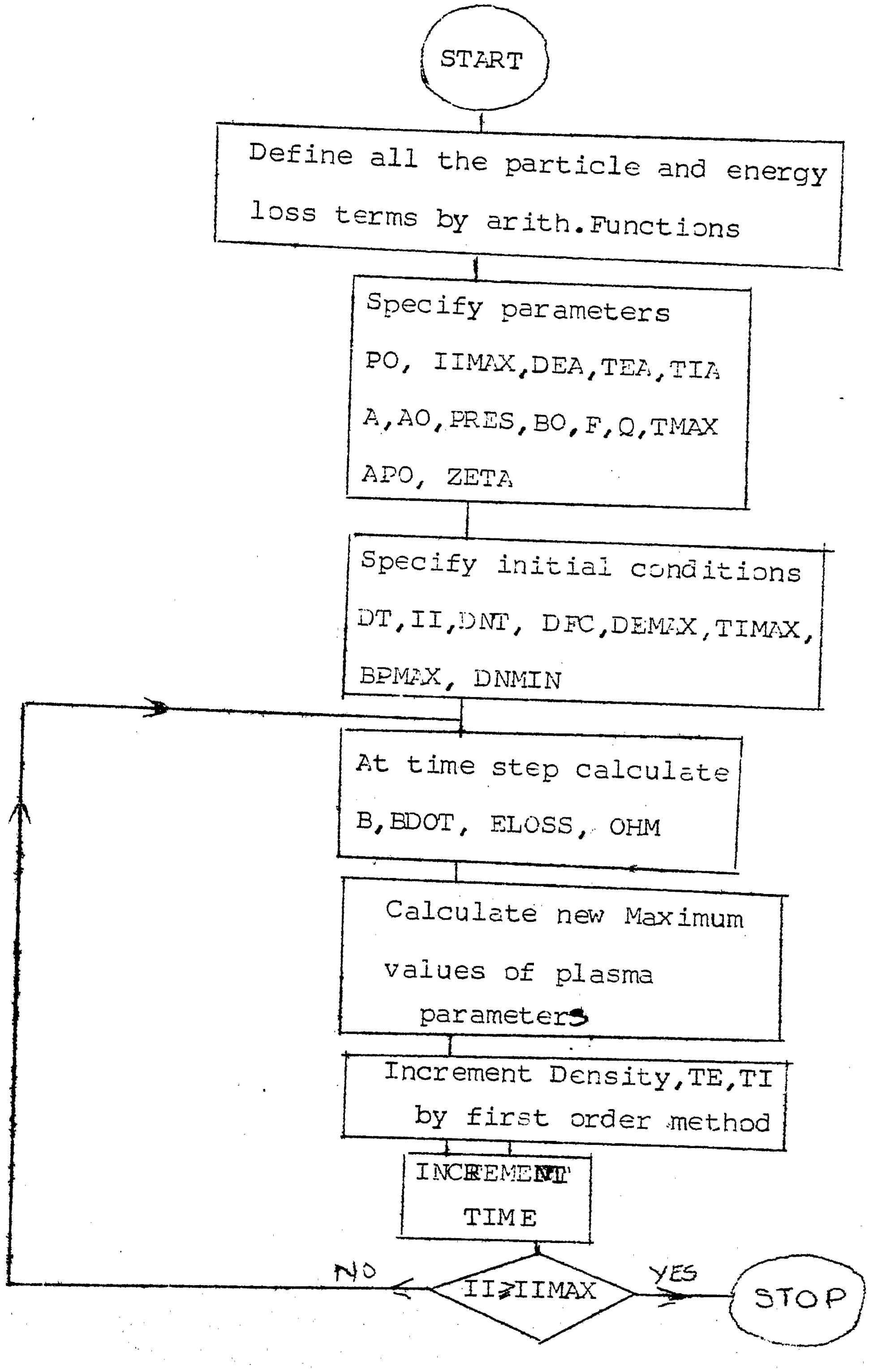
	BIGLEV	BIGUP	LITTLE	QUADX	PINCH
TMAX	0.043	0.043	0.005	0.002	0.005
A(cm^2)	50.0	50.0	18.0	6.0	18.0
AL	800.0	800.0	270.0	160.0	800
AO	0.0	700.0	90.0	3.0	90.0
QO	20000.0	20000.0	2000	500	2000

TEA the initial electron temperature , TIA the initial ion temperature and TMAX the duration of the experiment in seconds. We also specify the plasma radius , length of the plasma, the obstacle area, magnetic field BO, microwave cavity Q and the microwave frequencey F in GHZ. We specify the time step DT, the maximum neutral density DEMAX, the maximum electron temperature TEMAX, the maximum ion temperature TIMAX, the minimum value of neutral density DNM, the maximum value of the magnetic field the microwave power , the ion saturation current and the maximum value of energy loss.

We calculate the magnetic field and its derivative using the appropriate formula. The energy loss , BETA , the change in the neutral density DNT, are also calculated. After this the above calculations the current values of time DEA, TIA, B, P, SJI, ELOSS & BETA, are stored.

Next the densities of electrons, ions and neutral atoms are incremented using the first order method. Following this we check for the time taken and the program is allowed to run or terminate appropriately.

FLOW CHART


START
Define all the particle and energy loss terms by arith.Functions
Specify parameters
PO, IIMAX,DEA,TEA,TIA
A,AO,PRES,BO,F,Q,TMAX
APO, ZETA
Specify initial conditions
DT,II,DNT, DFC,DEMAX,TIMAX,
BPMAX, DNMIN
At time step calculate
B,BDOT, ELOSS, OHM
Calculate new Maximum
values of plasma
parameters
Increment Density,TE,TI
by first order method
INCREMENT
TIME
NO
II≥IIMAX
YES
STOP

3.3 SYSTEM PARAMETER ADJUSTMENT:-

MAGNETIC FIELD:- Any time dependent magnetic field can be specified. In this study we use

$$B(t) = B_o e^{-t/\tau} \sin \omega t$$

MICROWAVE POWER:- Any time dependent microwave power pulse can be specified. Reference (1) has used,

$$P = P_o \text{ MAX } (0, 2B/B_o-1)$$

Though the initial conditions could be set quite arbitrarily, . . in this study the initial conditions have been set according to references (1). A very interesting outcome of the simulation is regarding the independence of the plasma parameter variation with initial electron and ion temperatures on the one hand and their sensitivity the neutral pressures on the other.

3.4 ADDITIONAL PROGRAM FEATURES:- An interactive graphics package has been incorporated in the program to obviate the time consuming excercise of plotting the parameters on paper. In addition to the large amounts of time saved, it also ensures greater accuracy in flotting.

The program is adequately representated in the flow chart given in Figure (3.1).

CHAPTER-4

4.1 This study simulates a number of multipole devices and a theta-pinch device whose dimensions are given in table [3.1]. The first four devices correspond to multipole machines simulated by the Wisconsin group [1]. The pinch devices is similar to SCYLAC [2].

The intent of this chapter is to simulate the time variation of various plasma parameters like the temperature, density and Beta. With the help of the zero-di,ensional model of the magnetic confinement system obtained in Chapter 2, a detailed dynamic analysis of the system for various neutral pressures has been carried out. The main theme of this process is the prediction of the plasma scaling laws and determination of the parameter space in which the system may be made to operate. It is important here to recall the fundamental assumption associated with the zero-dimensional model, that all the variables in the computation can be space-averaged over the cross-section. The second assumption is that the resistivity of the plasma can be computed with the assumptionthat the plasma is fully ionized. This can be somewhat justified as one finds that the density of neutral atoms remains at a constant initial value though the ion and electron densities grow to 10-15 times the neutral density. The third assumption is that the plasma confinement time τ_p is a variable

dependent upon the interaction of neutral atoms with ions and electrons. The question of how correct are the above assumption is difficult to answer with any generality, and one is usually reduced to verifying the validity of the assumptions in various specific conditions. The sacrificein curred in the process thus limits its validity to a certain range of operating conditions. The most important aspect of these numerical studies is to determine whether all the experimental observations can be verified using the existing theory. For this we analyse the zero dimensional model by numercally solving the set of linear equations for predicting the transient behaviour. This qualitative comparision helps us in confirming the various types o f processes which are assumed to take place.

4.2 THETA-PINCH :

Fig.(4.2) is a plot of the time variation of the electron ion densities. The time variation of the electron or ion densities indicates that higher values of densities of electrons and ions are obtained at higher neutral pressures. This is due to large contributions of the ionization term. One also notices that the peak keeps on broadening, indicating a shift towards temperature equilibrium. There is also a noticeable shift in the peak or a delayed peaking with decreased neutral density indicating that the ionization term becomes effective only towards the end with the decrease of the loss terms.

Fig.(4.3) Is the time variation of electron temperatures. The time variation of electron temperatures indiciates the strikingly large values of temperatures that can be obtained using the pinch effect. The temperature variation also indicates a peak broadening indicating an early temperature equalization. This can be attributed the smaller number of neutral atoms present initially. The shifting of the occurance of the peak towards the starting point indicates the predominance of electron energy loss terms with increasing neutral pressures.

Fig. (4.4) Is the plot of time variation of ion temperatures. One observation which one can make on seeing the time variation of ion temperatures is that the was do not gain significant energy (_ 60eV). This is primarily due to the fact that the electrons being more mobile, absorb all the in coming microwave power and there is little time for ions to gain energy via electron ion collisions. One also notices that the maximum ion temperatures occur at neutral pressures which are neither too high nor too low. Rather they occur around temperatures 'which are' optimum' as will be seen later on.

Fig. (4.5) is the plot of the largest peak of 'BETA' in each of the simulation runs. Contrary to usual expectations, the BETA value goes through a minimum somewhere around the 'optimum' values of neutral pressures. The BETA values tend to be very large either for low neutral pressures or for high neutral pressures.

Fig. (4.6) is a plot the variation of the final plasma cross-section with various neutral pressure. The steep drop could be due to neumerical instability.

Fig. (4.7) is a plot of the plasma cross-sectional area as a variation with the operating time. The gradual increase in the cross-section is due to increased electron and ion densities and temperature resulting in decreased mean free path. The decrease in the cross-sectional area is due to decreased electron energies towards the latter half of the experiment. The final cross-sectional area is very law in sharp contrast to other values in the experiment primarily due to very law electron densities and temperatures. ~~This variation compares favorably with those in ref. [illegible]~~

Upon gradually varying the values of neutral pressures such that the initial neutral densities vary from 1.6×10^{12} to 6×10^{11} particles/cm^3 it was found that one could obtain reasonable variations of densities and temperatures only for neutral densities varying from 5.1×10^{11} to 5.8×10^{11} particles/cm^3. Upon assuming neutral densities either below 5.1×10^{11} or above 5.8×10^{11} particles/cm^3 the density variation or the electron temperature variation become insignificant. In this study neutral densities of 531×10^{11} particles/cm^3 was used as shown in fig. (4.2), (4.3), (4.4).

Fig. (4.8) is the plot of normalized values of plasma parameters with time at optimum neutral densities (531x10^{11}). One notices an early and broad electron temperature peak. This is because the microwave power input is primarily absorbed by the electrons initially. As predictable the magnetic field variation is a sinusoid peaking somewhere midway. The electron or ion densities shows a peaking in the latter help of the simultation. The iontemperatures show a peaking midway between the electron and density peaks. There is only a small space of time in which the ion temperatures, electron temperatures and densities show a peak. Significantly though the electron and density variations show a broad peak, the ion temperatures shows a sharp peak. The time lag of the B peak and density peak is noticable in the Wisconsin work (1).

OCTOPOLE :-

Fig. (4.9) is a plot of electron or ion density variation with time for various neutral densities. Unlike the theta-pinch , the density peaking occurs at the same location and there is apparently no shifting in the time of peaking. Secondly the peaking is also not as broad as that of the theta pinch. This is because the ionization term becomes predominant only midway in the simulation.

Fig. (4.10) is a plot of electron temperature variation with time for various neutral densities. One notices the increased maximum values of electron temperatures with decreased neutral pressures. There is also a predominant

tail coming into the picture at low neutral densities. This gives an indication that the losses which were predominantly due to Bremmstrahlung and synchrotron were significant only then. There is also a considerable amount of peak broadening indicating that ideally the electron temperatures attains a peak value and retains it till the end of the simulation.

Fig. (4.11) is a plot of ion temperatures varying ~~varying~~ with time. The ion gain temperature only in the latter half of the simulation indicating better electron-ion coupling then. In fact in the last few time steps the ion temperatures show an almost asymptotic increase in values. Even then the ions attain values of around (80eV) at very high neutral densities.

Fig. (4.12) is a plot of the variation of BETA with time at various initial neutral densities. It can be observed that maximum value of the BETA peak occurs at neutral densities which are around 17000×10^9 particles/cm^3. This slight deviation is due to the fact that (nT) product maximum occurs at this neutral density.

Similar to the theta-pinch, it was determined that neutral densities around 1.7×10^{12} particles/cm^1 were most suitable. Fig. (4.14) is a plot of normalized plasma parameters with time. Unlike the case of a theta pinch fig. (4.8) the electron temperatures and densities not have a broad peak. As one would have also noticed the maximum values of the various parameters

are several orders less than those corresponding to the theta pinch. Also present in the ion and electron temperature variation is the tail which attains very large values towards the end of the experiment. This is primarily due to reduced losses towards the end of the experiment.

Ref. (1) has indicated that the energy loss rate can be taken to be proportional to -dB/dt. Upon incorporating such term one can obtain the plot of energy loss rate varsus time as shown in fig. (4.13) . Ref.(1) has indiciated that this can be taken to be the rate at which the magnetic field lines leave the system. Fig.(4.13) indicates that the energy loss rate grows rapidly with time reaching very large values towards the end of the experiment. There is no energy loss in the first half of the experiment because here the magnetic field lines grow stronger with time.

THEORETICAL BASIS FOR NEUTRAL PRESSURE VARIATION :

In this study the pinch formed after the pre-ionization. Although heating is not a problem for the high energy injection device, a new difficulty arises. The magnetic region in which the injected ions are trapped contains a large number of neutral gas atoms, owing to the finite initial pressure which any system must have. The neutral gas produces a steady loss of the trapped ions by the process of charge exchange. In this process the hot ion captures an electron from the

gas atom and thus escapes from the system as an energetic neutral atom. Thus charge exchange competes quite seriously with the desired nuclear reactions in the gas.

It would thus seem that the first major barrier which must be overcome by a high energy injection device is the neutral gas background.

DIRECT (BRUTE FORCE) PUMPING :

The most obvious method, of course, is simply to reduce the initial pressure of the system until charge-exchange no longer competes with the usual plasma processes of Coulomb scattering and nuclear reactions. The loss rate of ions/unit volume can be shown to be limited by the charge exchange.

NEUTRAL ''BURNOUT'':

It is possible to obtain a condition for creation of a thermonuclear plasma which is less restrictive than the one above. This takes advantage of the fact that the self shielding action of the plasma tends to lower the neutral atom density in the plasma interior. Neutral atoms are removed in two ways. One is by the charge-exchange process itself since the fast neutral which leaves the plasma region. The second and more effective, process is simple ionization of the neutrals by the fast ions (and electrons) of the plasma.

As long as the rate of destruction of neutrals in the

plasma interior is small than the rate of influx of neutrals from external vacuum region, there will be no appreciable difference between the neutral density internal to the plasma and external to the same. There is a critical point, however for the injected current at which point the neutrals are being destroyed as soon as they enter the plasma region. Once this point is passed, the neutral density drops in the interior and the plasma density increases, hence burningout more ''neutrals''. This critical point will be called ''burnout'' . It is expected that the neutral density will drop sharply from its vacuum value when the injected current is only slightly above the burnout point.

A crude first estimate of the burnout current is obtainable directly from the considerations above.

TRAPPED ION PUMPING (GETTERING) :

A certain fraction of the neutral atoms of the system become ionized upon passage through the trapped plasma region. These ions are then constrained to move along a field line and will drift out to the ends of the mirror container. If suitable pumping arrangements are then provided, this mechanism could represent an important addition to the total pumping speed. In some cases it may dominate and appreciably lower the entire pressure in the interior of the system. In this case the trapped ions, of course, are acting exactly as an ion-pump, or as a getter.

It has thus been demonstrated that several aspects of the dynamics of pinch formations can be brought out using even a zero dimensional model of the pinch device.

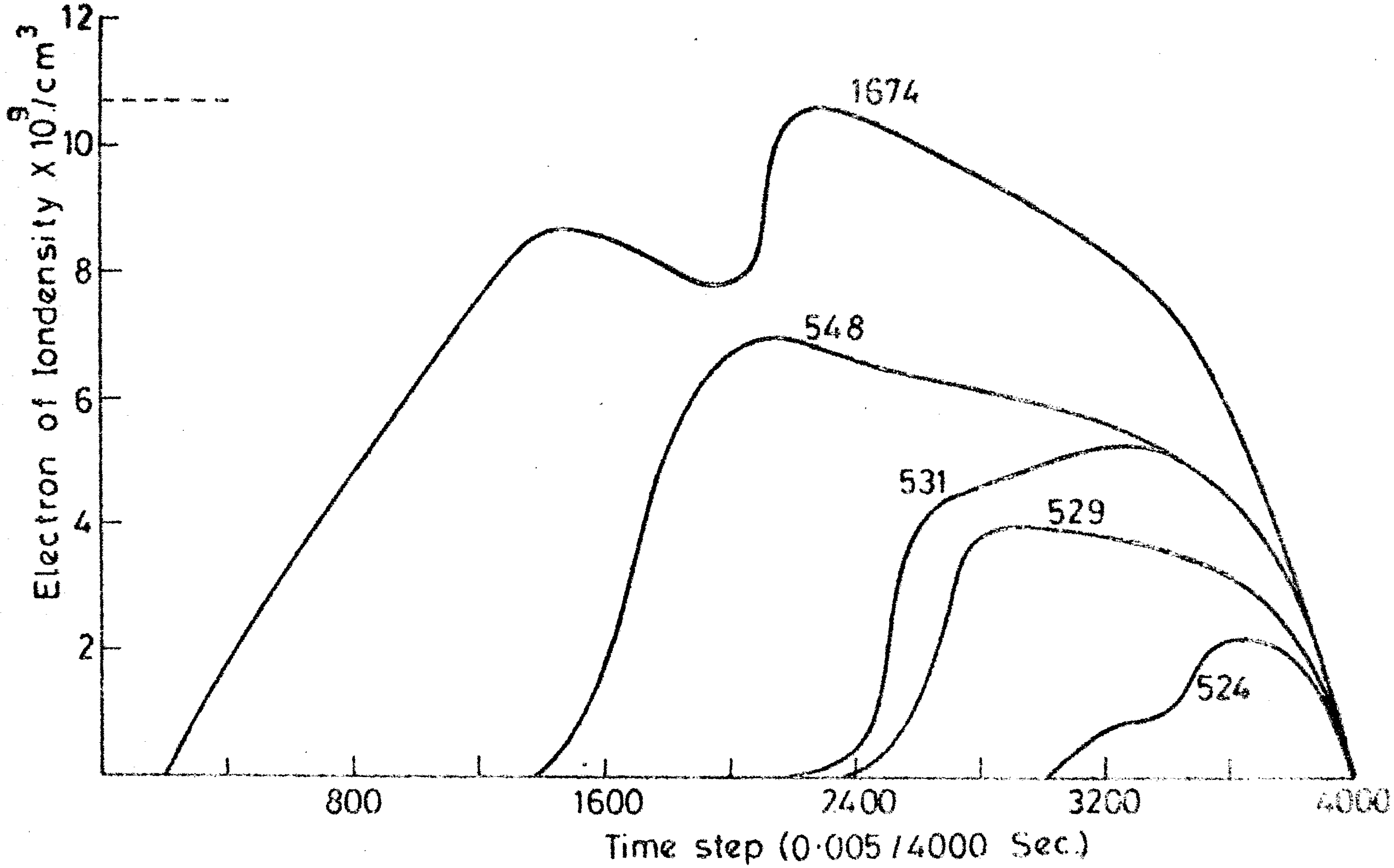


FIG. 4·2 PLOT OF ELECTRON OR ION DENSITY VS TIME

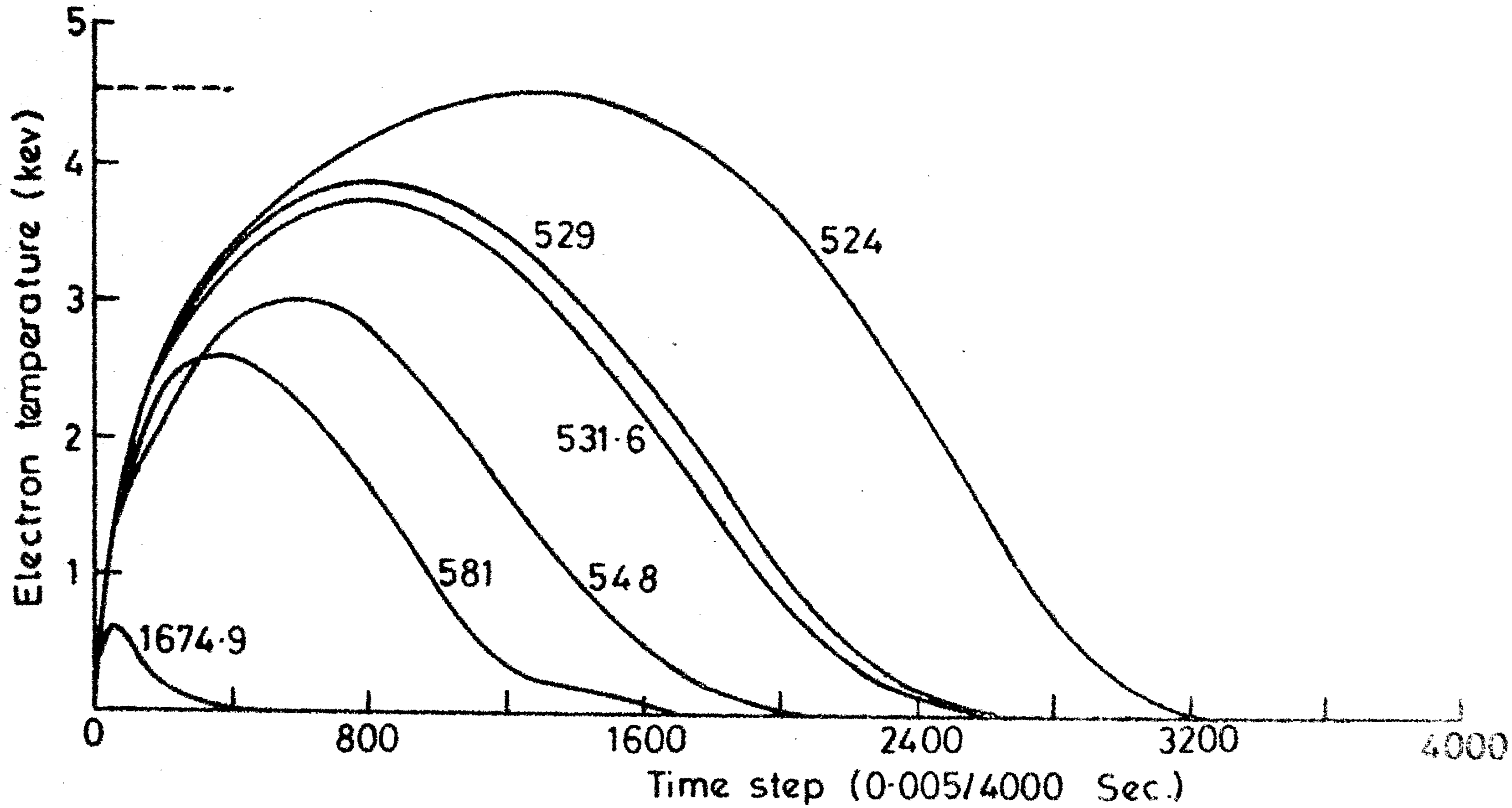


FIG. 4·3 PLOT OF TE VS TIME

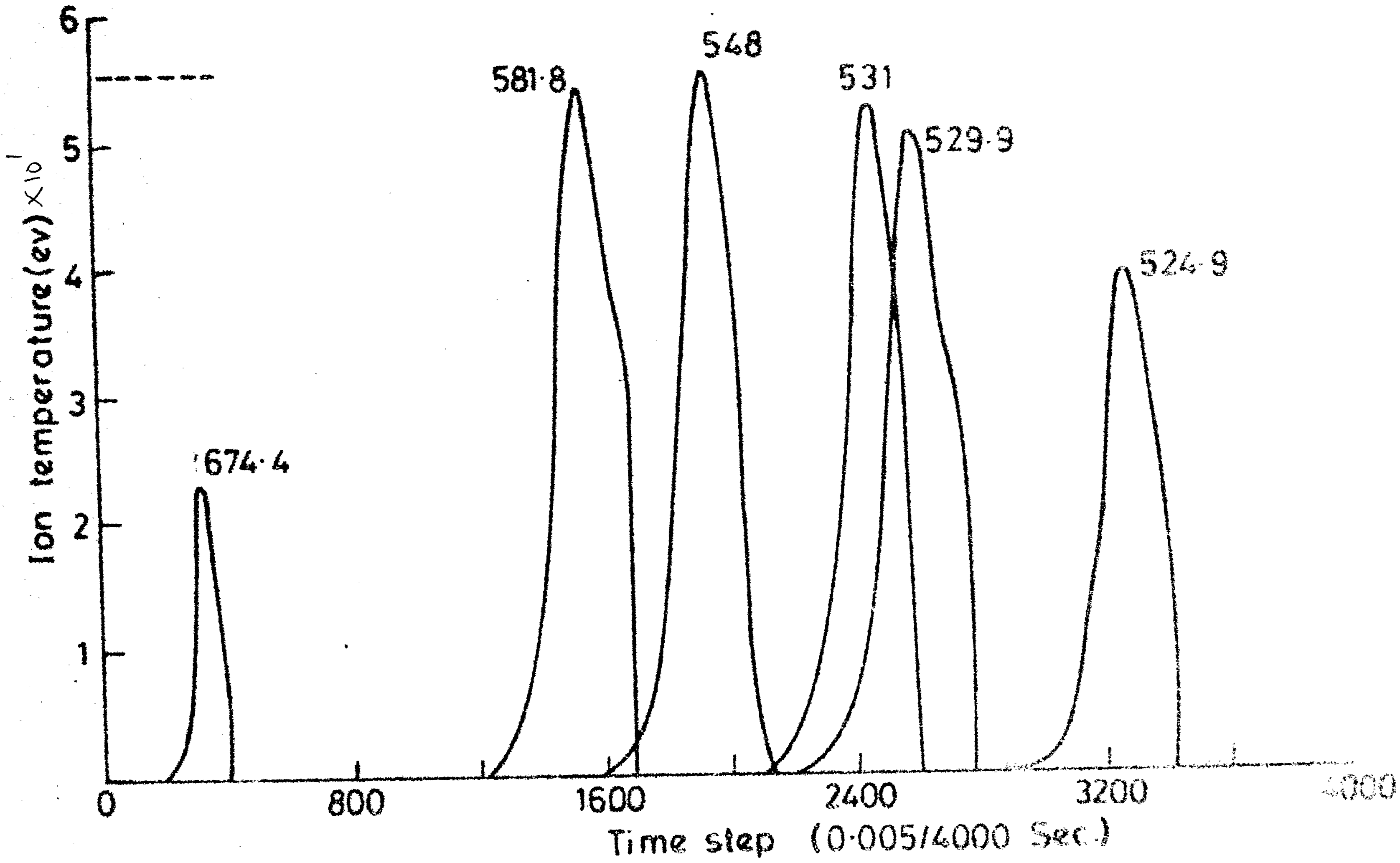


FIG. 4·4 PLOT OF TI VS TIME

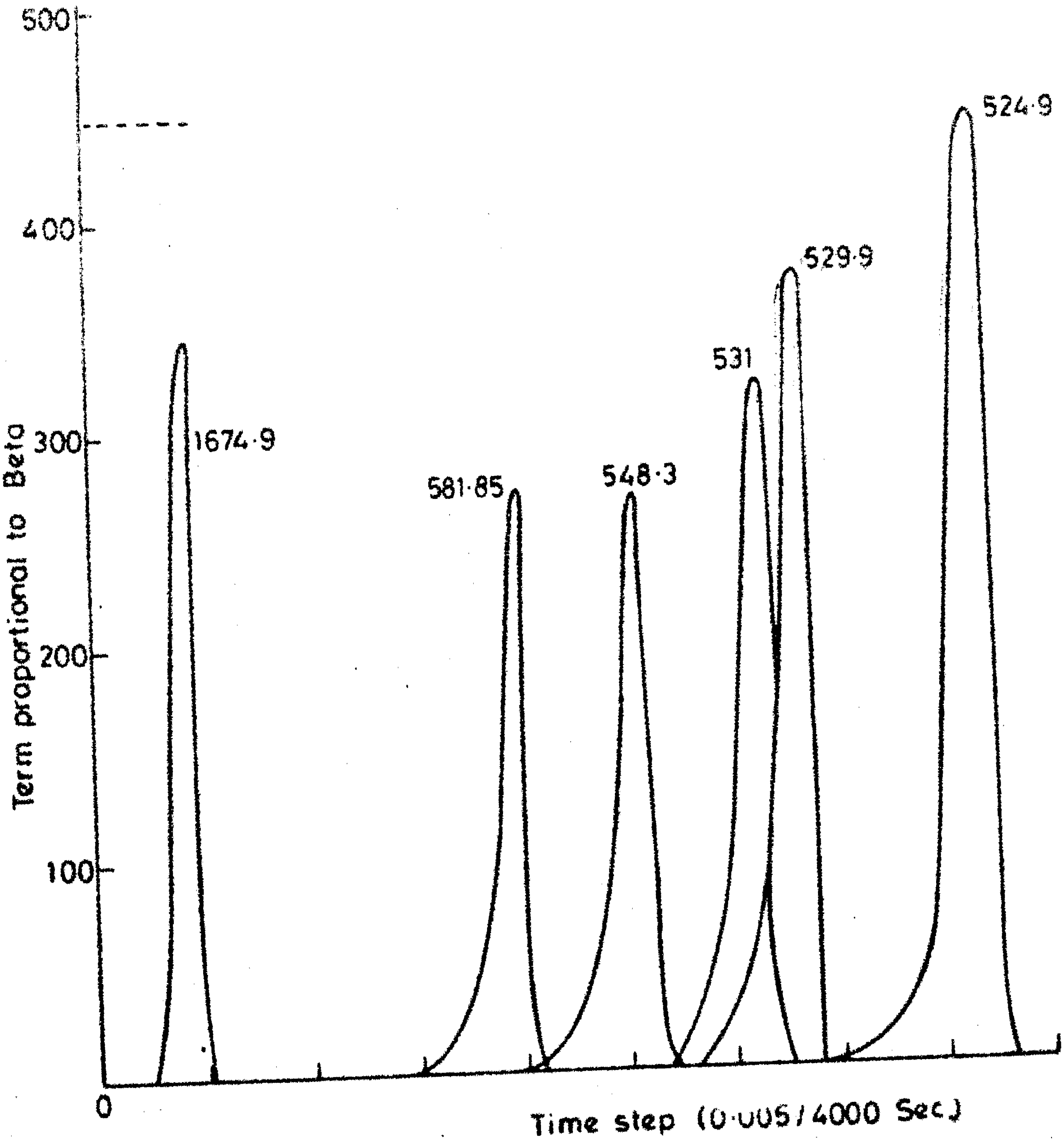


FIG. 4·5 PLOT OF β VS TIME

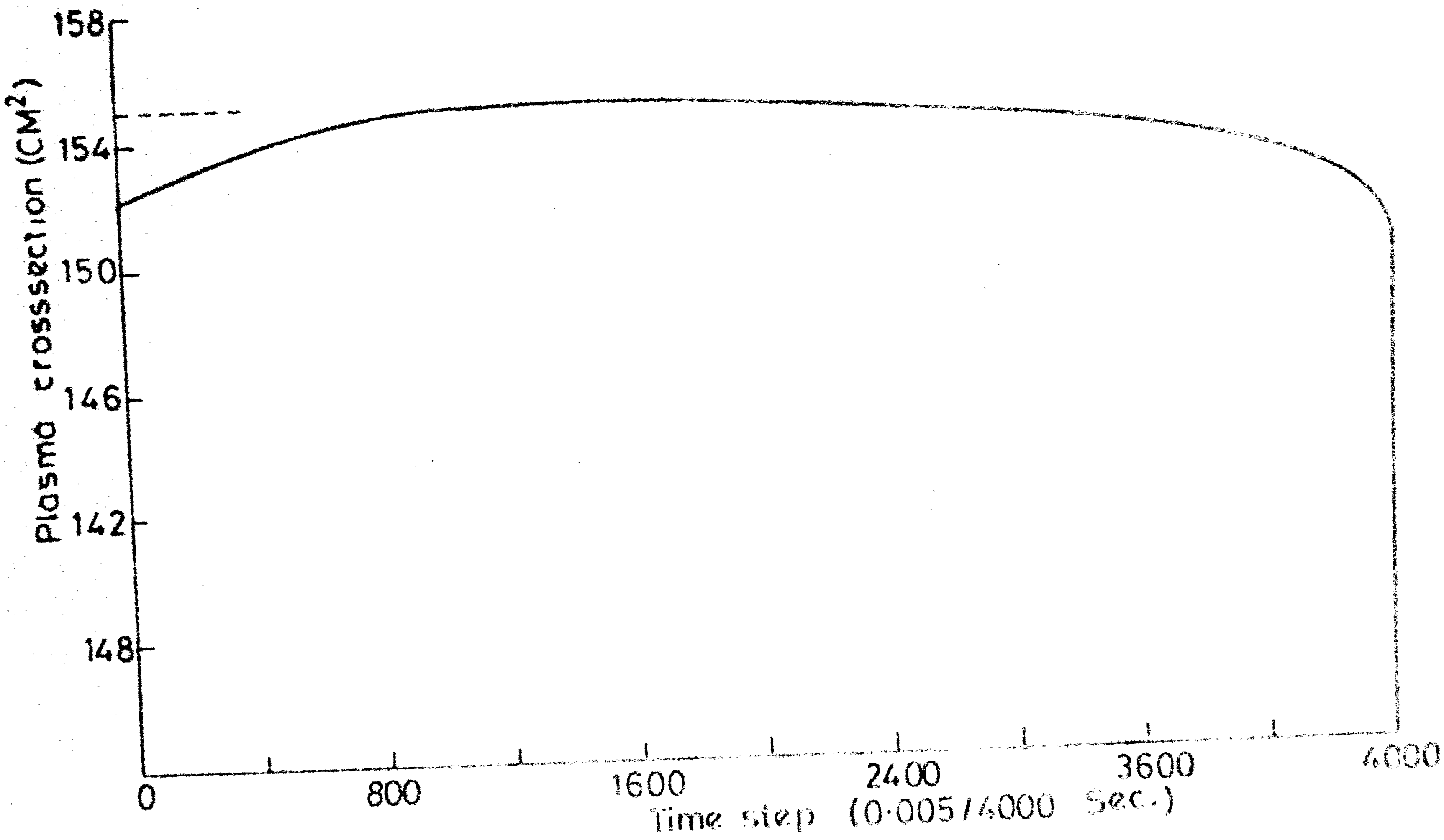


FIG. 4·7 PLOT OF AREA (PLASMA) VS TIME

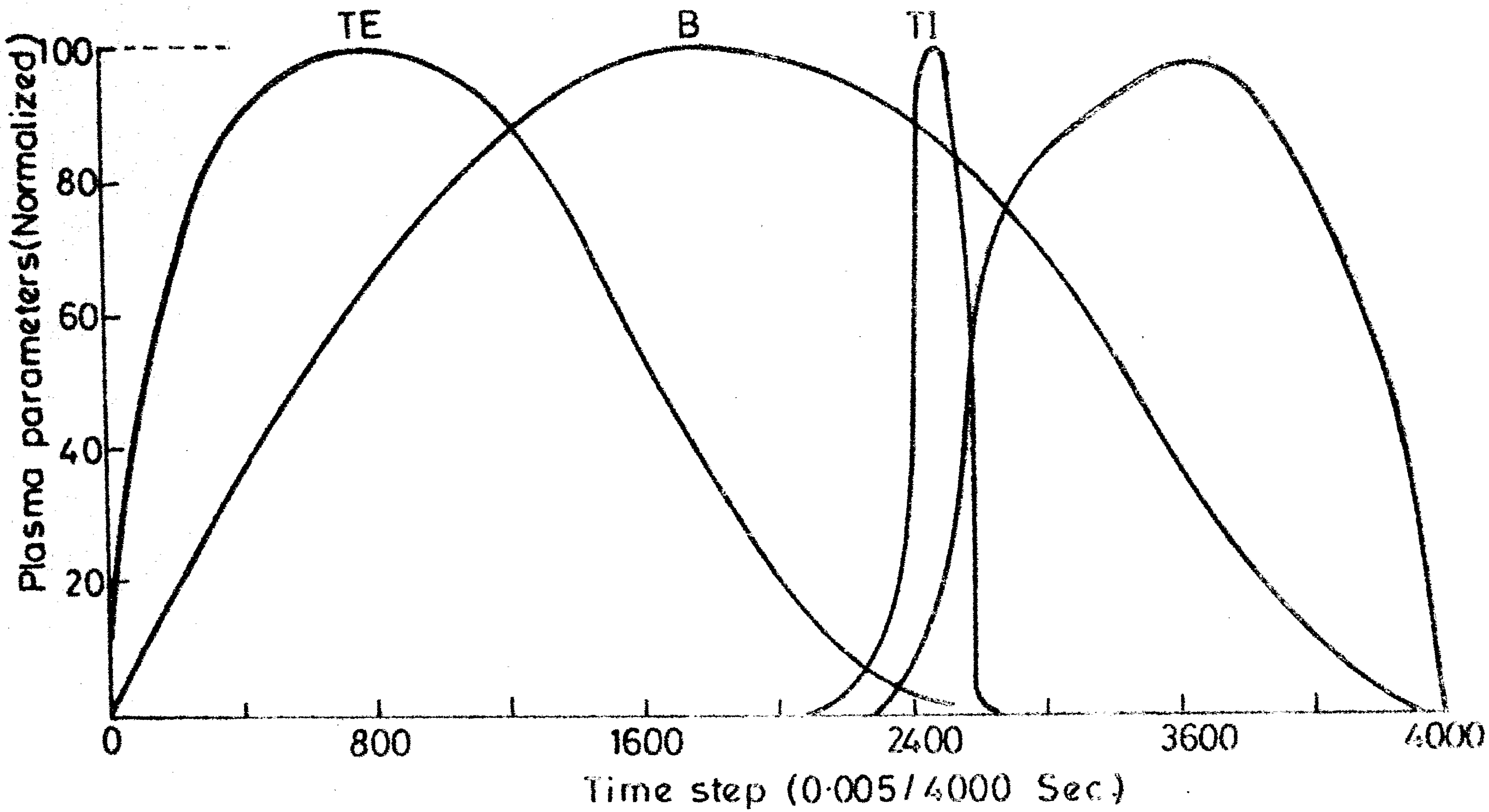


FIG. 4·8 PLOT OF NORMALIZED PLASMA PARAMETERS VS TIME θ-PINCH SIMULATION

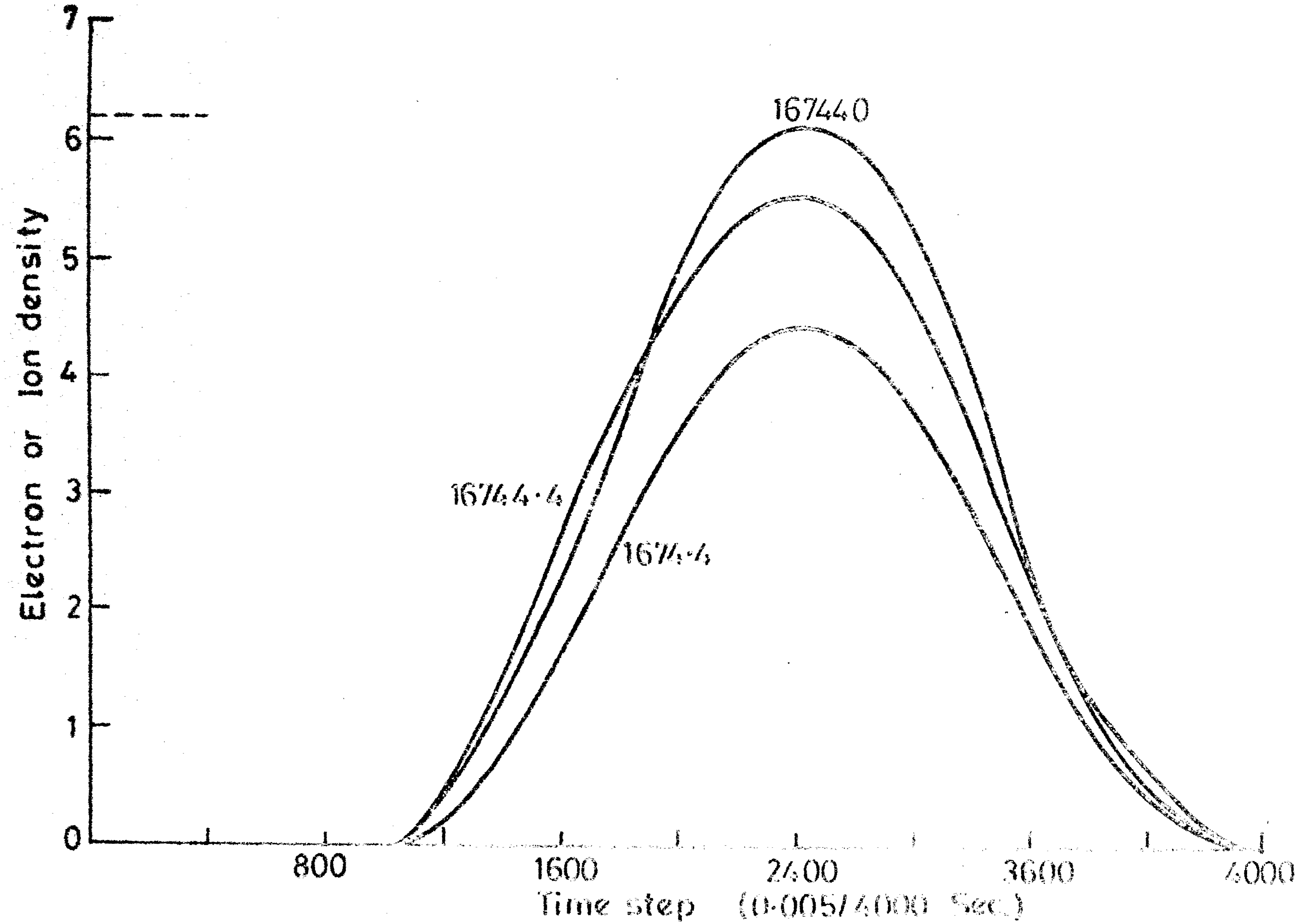


FIG. 4·9 PLOT OF DENSITY VS TIME FOR VARIOUS PRESSURES

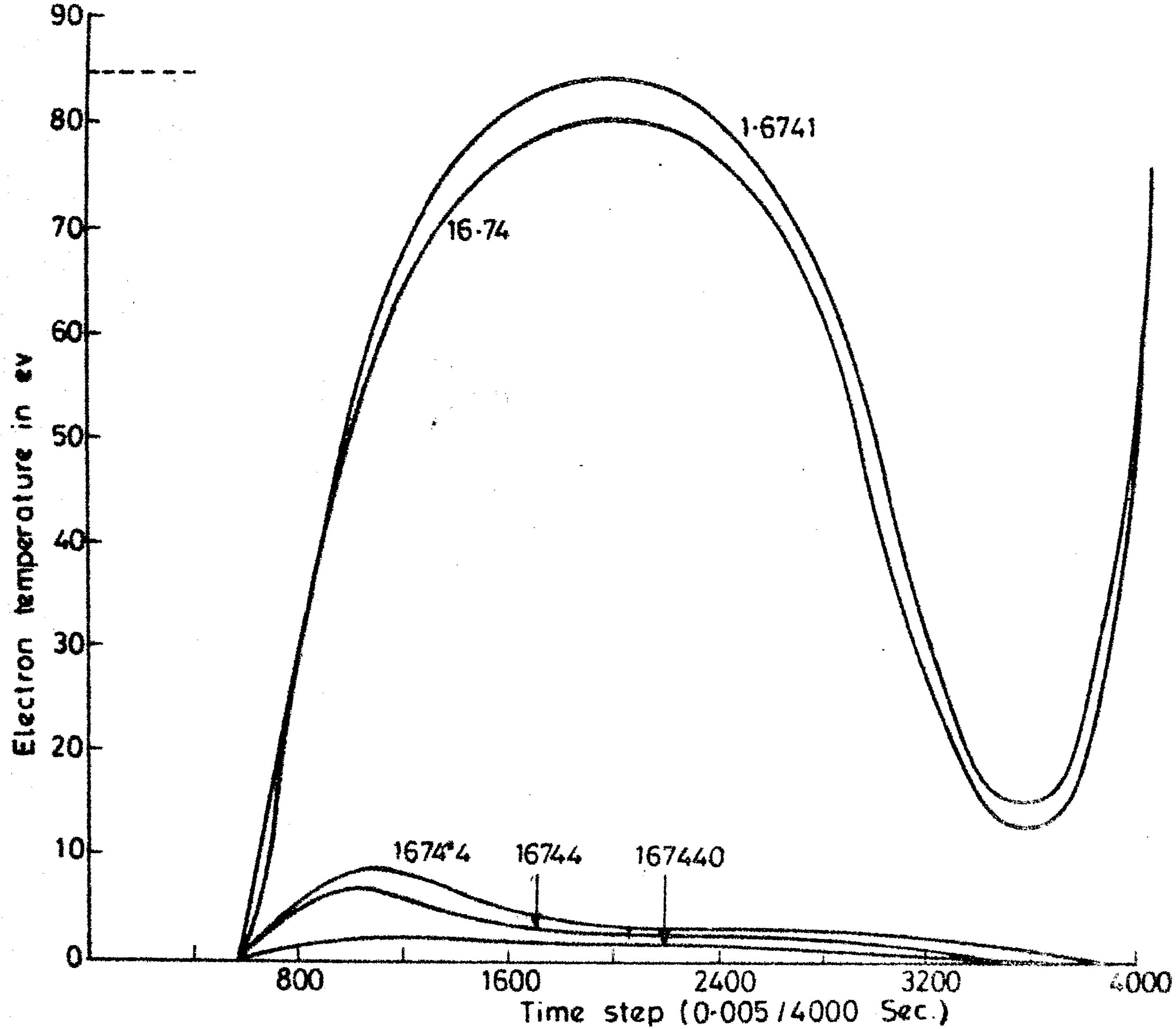


FIG. 4·10 PLOT OF TE VS TIME FOR VARIOUS PRESSURES

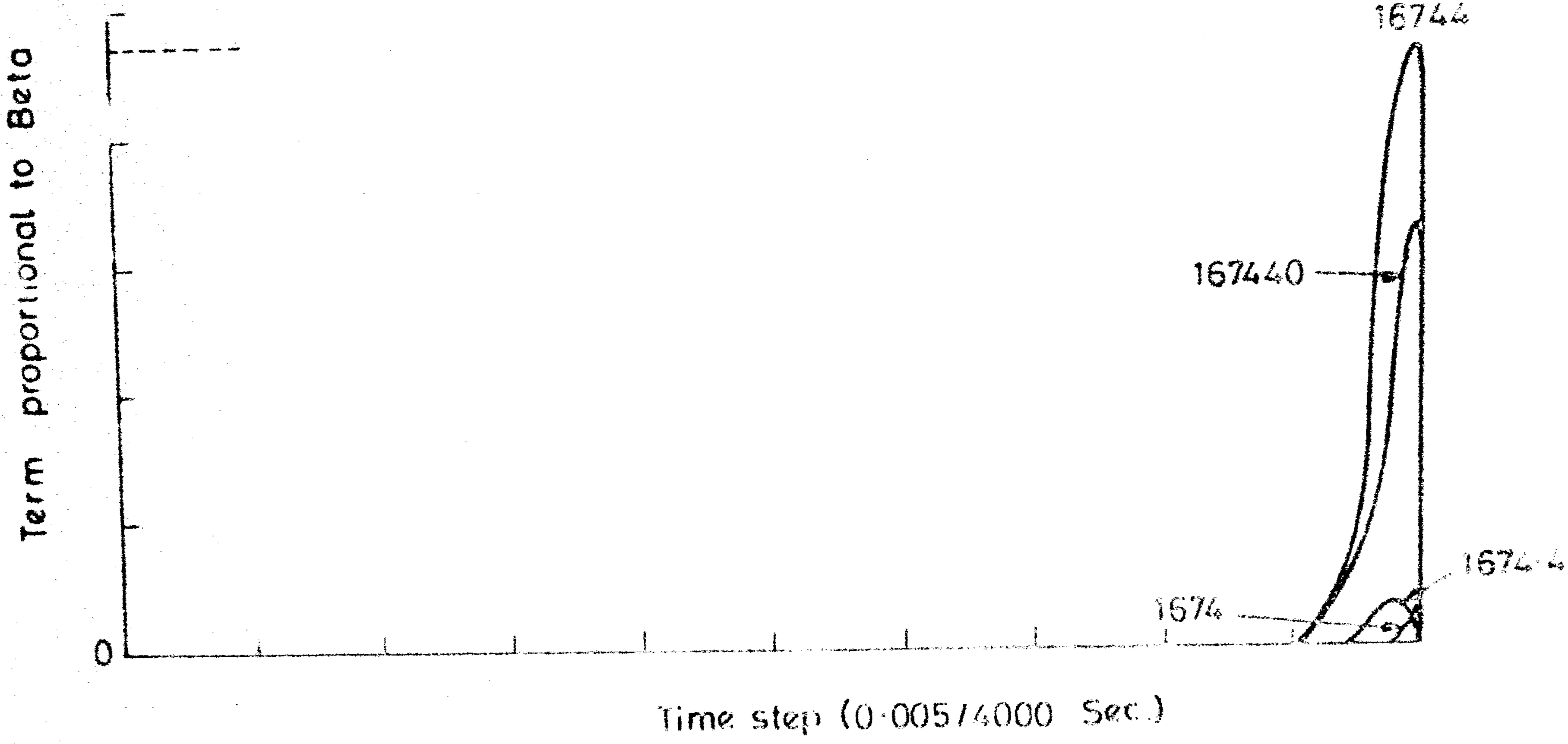


FIG.4·12 PLOT OF BETA VS TIME FOR VARIOUS PRESSURE

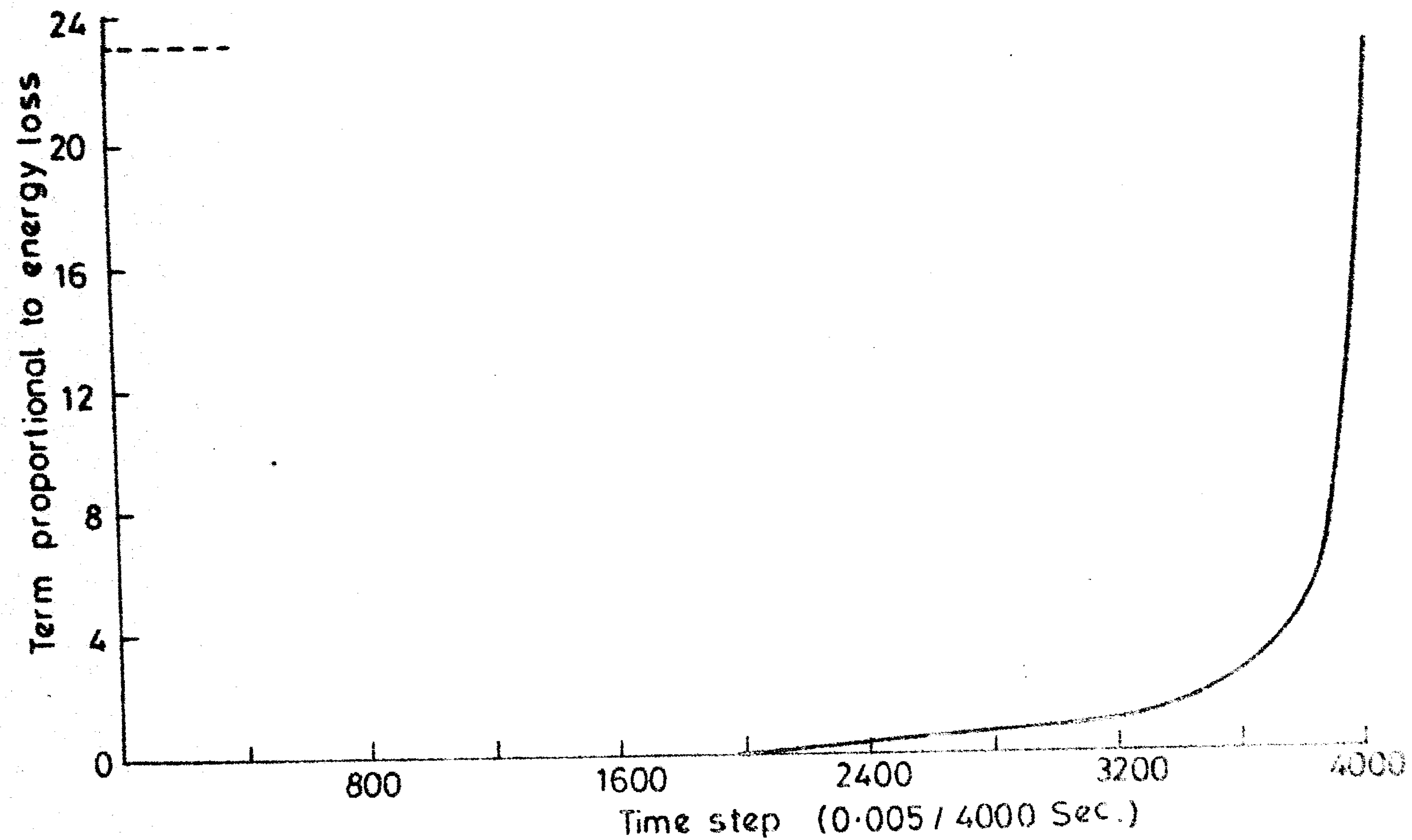


FIG.4·13 PLOT OF ELOSS RATE VS TIME (OCTOPOLE AND θ-PINCH)

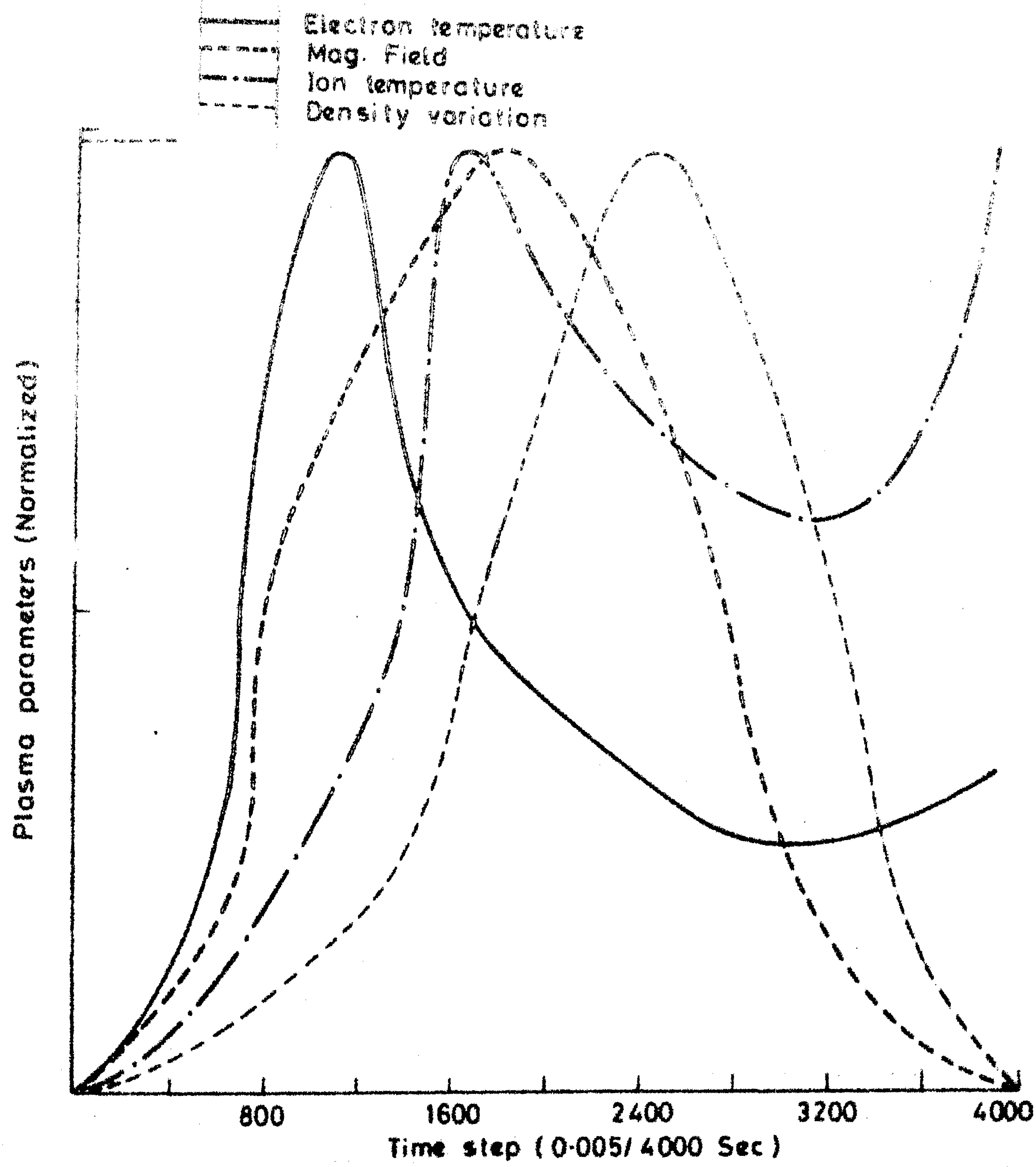


FIG. 4-14 PLOT OF NORMALIZED PLASMA PARAMETERS VS TIME OCTOPOLE SIMULATION

CHAPTER-5

5.1 SUMMARY AND CONCLUSIONS:

In the present work we have developed a zero-dimensional model of a octopole and a theta-pinch device for simulating the dynamic behaviour and properties of various plasma parameters.

Chapter 1 is devoted to a general description and some useful definitions related to the present work. It also contains literature review and outline of the present work.

In Chapter 2 we obtained zero-dimensional models of a theta-pinch device and a octopole by deriving the various energy loss or gain terms. Our aim was to study the time behaviour of various plasma parmaters under various initial operating conditions, such that one could determine if possible parameter spaces of feasible and stable operation.

A very acceptable variation plasma densities and temperatures is is obtained at neutral densities between 580×10^9 and 510×10^9 particles / cm^3 for a theta pinch. If the neutral densities are higher than 580×10^9 particle/cm^3 the variation of temperature s becomes very predominant with

the electron and ion densities tending to be very low. The variation of the plasma cross-sectional area is also very typical of a theta-pinch device. The variation in temperature and densities for the octopole is several orders less compared to a theta-pinch device for a given set of initial conditions.

5.2 FURTHER SUGGESTED WORK:-

The first improvement that can be done concerns the method adopted to solve the equations of the zero-dimensional model. In this study a first order method has been used. More accurate results can be obtained if the equations were solved using a second order method (Appendix A). J.R.McCowan et al (13) have indicated in a paper that Heindrich et al (14) found the rotational energy to be comparable to that of the thermal energy. Hence the code could be modified in order to be able to examine possible effects of rotation on transport phenomena in the plasma. The zero D model includes five first order differential equations. Four of the equation are derived from expressions for the time rate of change of electron and

ion temperatures, total particle in ventory and the magnetic flux within the plasma. The fifth equation is obtained from radial pressure balance.

Another extension could be in the direction of developing a zero-D model of a reversed fieldpinch. Here in this type of fusion device, the closed field lines are produced internally. A progress report on "Study of plasma convection and wall interactions in magnetic confinement system" which is a US department of energy project briefly explains the recently developed zero-D model.

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APPENDIX A

The three equations derived in Chapter 2 can be solved simultaneously by the following second order method. Given a time step dt and the time derivation $\dot{n}$, T_e, T_i at time t, then calculate n, T_e, T_i at time t+dt. First calculate B, P, n_o at time t. Second increment n, T_e, T_i to approximate them at t+0.5 dt (x is any of n, T_e, T_i):

$$x(t+0.5dt) = x(t) + (0.5dt)(x_{t+0.5\,dt}).$$

Third, calculate n, U_e, U_i at t+0.5dt, using values of B, P, n_o at time t and n_e, T_e, T_i at t+0.5dt. Fourth, calculate T_e, T_i from $T_{e,i} = (\frac{2}{3} U_{e,i} - T_{e,i}\, n)/n.$

Finally, using n, T_e, T_i at t+0.5dt, increment the original n, T_e, T_i at time t to get (x is any of n, T_e, T_i)

$$x(t+dt) = x(t) + (dt)(\dot{x}_{t+0.5dt})$$

APPENDIX C

Calculation of $\frac{\partial \emptyset}{\partial t}$

Using Maxwell's equations,

$$\nabla \times B = \mu_o J$$

$$\nabla \times \varepsilon = \frac{B}{t}$$

Consider now the magnetic field diffusion equation. The plasma magnetic field $B_i(r)$ is related to the external magnetic field, plasma column area, and resistivity by a magnetic field

diffusion equation obtained from. Maxwell's equation and Ohm's law in the form

$$J_{\theta} = (1/\eta_{\perp})(E_{\theta} + U_r B_i)$$

where E is the azimuthal electric field and U_r is the plasma radial velocity. Since the plasma diamagnetic current flows transverse to the magnetic field lines and since the magnetic field in the sheath is of sufficient magnitude so that the electron gyrofrequency is many orders of magnitude larger than the electron- ion collision frequency, the strong magnetic field limit expression for plasma resistivity is used in this analysis [4]

$$\eta_{\perp} = 3.27 \times 10^{-9} \ln \ /T_e^{3/2} .$$

Reference [4] indicates that the magnetic field equation can be expressed as

$$\frac{\partial}{\partial t} = \nabla \times \left(\frac{\eta_{\perp}}{\mu_o} \nabla \times \bar{B} - \bar{U} \times \bar{B} \right)$$

With the long compression oil length assumption, Br/Z <<1, the z-component can be written

$$\frac{\partial B_Z}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r}\left(\frac{\eta_{\perp}}{\mu_o} \frac{\partial B_Z}{\partial r}\right) - \frac{1}{r} \frac{\partial}{\partial r} (rU_r B_Z)$$

Integrating the above equation over the plasma cross sectional area and using the Leibnitz's rule for differentiating integrals

$$\frac{\partial}{\partial t} \int_{Ap} B_Z d - B_Z \frac{\partial A_p}{\partial t} = \int_{Ap} \frac{1}{r} \frac{\partial}{\partial r}\left(\frac{\eta_{\perp}}{} r \frac{\partial B_Z}{\partial}\right) dA$$

$$- \int_{Ap} \frac{1}{r} \frac{\partial}{\partial r}(rU_r B_Z) dA$$

Using $A = \pi r^2$ and $U_r = (\partial A/\partial t)(1/2\pi r)$, then the second terms on both sides cancel leaving

$$\frac{\partial \emptyset}{\partial t} = \int_{Ap} \frac{1}{r} \frac{\partial}{\partial r}\left(\frac{\eta_\perp}{\mu_o} r \frac{\partial B_r}{\partial r}\right) dA$$

$$= 2 \quad a \frac{\eta_\perp}{\mu_o} \frac{\partial B_Z}{\partial r} \Big|_{r=a}$$

```
[illegible]
[illegible] (101),[illegible](101),[illegible](101),[illegible](101),[illegible](101)
[illegible] (101),[illegible](101),[illegible](101)
[illegible] FUNCTIONS
[illegible]=((((6*S/R**2)*(1.0/12.57E-7))**2)*(3.27E-9*[illegible]
*((TEE*1.0E-3)**3)))*AL*2.0/(22.077.0)*X
D1 IS DN/DT DUE TO IONIZATION
D1(DENS,DNUET,TE)=50.4*DENS*DNUET*EXP(-17.0/(TE+0.3))/TE**0.125
D2 IS DN/DT DUE TO DIFFUSION
D2(DENS,TE)=DENS*(0.33*DENS/SQRT(TE)+1.9E-[illegible]*DNUET*TE)/(R*R*B*B)
D3 IS DN/DT DUE TO OBSTACLE LOSSES
D3(DENS,TE)=2.0E5*DENS*AO*SQRT(AMAX1(TE,TIA))/(R*R*AL)
PE1 IS DUE TO MICROWAVES
PE1(P)=2.[illegible]*P/((1.0+DENG/DEA)*(R*S*AL))
PE2 IS DUE TO ION COLLISIONS
PE2(DENS,TE,TI)=61.0*DENS*DENS*(TE-TI)/TE**1.5
PE4 IS DUE TO BREMSSTRAHLUNG
PE4(DENS,TE)=1.7E-4*DENS*DENS*SQRT(TE)
PE5 IS DUE TO SYNCHROTRON RADIATION
PE5(DENS,TE)=3.87E-3*DENS*B*B*TE*(1.0+TE/2.04E5)
PE6 IS DUC/DT DUE TO THERMAL CONDUCTION
PE6(DENS,TE)=2.5*TE*D2(DENS,TE)+5.5*TE*D3(DENS,TE)
PE7 IS DUE/DT DUE TO PLASMA CONTRACTION
PE7(TEA,ADDTE)=(DEA*TEA*1.0E-3*AL)/ADDTE
PI3 IS DUI/DT DUE TO CHARGE EXCHANGE
PI3(DENS,DNUET,TI)=0.0186*DENS*DNUET*TI**3/(TI+100.0)
PI5 IS DUI/DT DUE TO THERMAL CONDUCTION
PI5(DENS,TE,TI)=2.5*TI*D2(DENS,TE)+2.0*TI*D3(DENS,TE)
PI7 IS DUI/DT DUE TO PLASMA CONTRACTION
PI7(TIA,ADDTE)=(DEA*TIA*1.0E-3*AL)/ADDTE
SPECIFY PARAMETERS
PO IS PEAK MICROWAVE POWER IN WATTS
PO=10.0
CONTINUE
ITMAX IS NUMBER OF ITERATIONS
DEA IS INITIAL DENSITY IN 10**9/CC
TEA IS INITIAL ELECTRON TEMPERATURE IN EV
TIA IS INITIAL ION TEMPERATURE IN EV
AREA [illegible] IN [illegible]
AL IS LENGTH IN CM
AO IS OBSTACLE AREA IN SQ CM
PNEUT IS NEUTRAL PRESSURE IN 10**-5 TORR
B IS FIELD AT [illegible] IN KGAUSS
```

```
C      THE MICROWAVE CAVITY(0)
C      ... IS DURATION OF EXPERIMENT IN SECONDS
C      ... CMS INITIAL PLASMA RADIUS
C      ZETA IS THE RATIO DA/DE
C      EPSE IS THE ENERGY LOSS PER ELECTRON
C      EPSI IS THE ENERGY LOSS PER ION
C      AVTCHE IS THE THERMAL CONDUCTION TIME FOR ELECTRON
C      AVTCHI IS THE THERMAL CONDUCTION TIME FOR ION
C      H DEFINES THE STEP FUNCTION H(0.25R-LAMBI)
C      LAMBI IS THE ION-ION MEAN FREE PATH
       ZETA=1.5
       AVTCHE=0.0
       AVTCHI=0.0
       IIMAX=1000
       DEA=1.0
       TEA=1.0
       TIA=0.025
       R=0.8
       APO=22.0*R*R/7.0
       PRES=5.0E-2/31.75
       AP=APO
       AM=300.0
       AO=9.0
       BO=A5
       F=2.45
       D=20...
       TMAX=0.5
       B=1.0E+5
C      SPECIFY INITIAL CONDITIONS
       TAU=0.33*A/(B**2*AP**2*TEA**0.5)+1.0E-3*CHUEZ*TEA/(B**2*AP**2)
       TAU=TAU+2.035*AO*SQRT(TEA+TIA)/(AP**2*AO)
       TAU=1.0/TAU
       TIME=0.0
       IPRINT=IIMAX/100
       DT=TMAX/FLOAT(IIMAX)
       IT=0
       DAT=322.22*PRES
       DFC=12.86*PRES
       DEMAX=DEA
       TEMAX=TEA
       TIMAX=TIA
       DOMAX=335.0*PRES
       ...MAX=...
```

```
GAMMA=?.?
?Z=1.?
RECORD VALUES
WRITE(?,400)
FORMAT(1X,'        STOP          TIME              DENSITY
2          ??           ONSET              FIELD')
CONTINUE
SUM=0.0
N=40
BB=SQRT((AP/22.0)/7.0)
H=(BB-0.0)/FLOAT(N)
SUM=F3(0,TEA)+F3(BB,TEA)
NN=N-1
DO 111 K=1,NN
SUM=SUM+2.0*F3((A+FLOAT(K)*H),TEA)
CONTINUE
DUM=H*SUM/2.0
BOLD=B
B=B0*SIN(3.1459*TIME/TMAX)*EXP(-0.5*TIME/TMAX)/0.7807+1.0E-5
B=SQRT(ABS(B*B-4.05E-8*DEA*(TEA+TIA)))
BDOT=(B-BOLD)/DT
ENERGY LOSS RATE CALCULATED
ELOSS=1.0*AMAX1(0.0,-BDOT)/B
BETA CALCULATED
BETA=(DEA*(TIA+TEA)*2.0*12.57E-7*1.6E+3)/(B*B)
BETA=(DEA*(TIA+TEA))*1000.0/(B**2)
BETA1=(16.0*22.0/7.0)*(TEA+TIA)*1000.0/(B**2)
EPSE=2.5*TEA*1.0E-3
EPSE=0.0
EPSI=2.5*TIA*1.0E-3
EPSI=0.0
RES=3.27E-9*2.2/SQRT((TEA*1.0E-3)**3)
DEL=2.0*(EPSE+EPSI)/(3.0*TA0)+TEA*(R/DTIME)
GAMI=0.25*AU
MUO=12.57E-7
I=0.0
ADOTE=-(2.0*BDOT/B)*5.0/(5.0+ZETA**2)-(TEA+TIA)*1.0E-3)
ADOTE=ADOTE-3.0*(1.0-ZETA**2)*DEL/((5.0+ZETA**2)*(T/B)**0.66667)/0
DEG=EXP*(0.0045*(T/B)**4+37.0*273.0*(T/B)**0.66667)/0
P=AMAX1(0.0,2.0*PO*(B/B0-0.5))
SI=DNI*DEA*SQRT(AMAX1(TEA,TIA))
DV=0.0
DPC=DEC
   *DPLAS*EXP(-5.07E-4*DE..*A*EXP(-17.0/(TEA*1.3)))/
```

```
[illegible]
[illegible](-1.7E-[illegible]/[illegible](-17.[illegible])/
[illegible])
[illegible] DTE=[illegible]EXP(-1.[illegible]*DT/R)
[illegible] DEC=DEC*(DEC([illegible])*EXP(-3.[illegible](DT/R)
[illegible]
DEMAX=AMAX1(DEMAX,DEA)
TEMAX=AMAX1(TEMAX,TEA)
TIMAX=AMAX1(TIMAX,T(R))
DDMIN=AMIN1(DDMIN,DSOET)
BPMAX=AMAX1(BPMAX,B)
PPMAX=AMAX1(PPMAX,P)
SDMAX=AMAX1(SDMAX,SDT)
IF (MOD(II+1,IPRINT).NE.1) GO TO 400
WRITE (22,460) II,TIME,DEA,TEA,TIA,DNSET,B,[illegible]
WRITE(24,[illegible])EDOSS,[illegible],TIME
FORMAT(2X,F13.4,2X,E23.11,2X,F13.4)
WRITE(23,900) AP
FORMAT(2X,F13.[illegible])
FORMAT(1H,5F13.4)
FORMAT(1H ,I13,6F13.4,3X,E16.4)
I=II/IPRINT+1
T(I)=TIME
DE(I)=DEA
TE(I)=TEA
TI(I)=TIA
BP(I)=B
PP(I)=P
SJ(I)=SJT
CONTINUE
IF(II.GE.IIMAX) GO TO 600
INCREMENT DENSITY
DDENS=D1(DEA,DNSET,TEA)*DT-D2(DEA,TEA)*DT-D3(DEA,TEA)*DT
IF (ABS(DDENS).GT.0.5*DEA)DDENS=SIGN(0.5*DEA,DDENS)
DEA=DEA+DDENS
INCREMENT ELECTRON TEMPERATURE
DTE=-30.0*D1(DEA,DNSET,TEA)*DT,
1-P2(DEA,TEA,T(R))*DT
2-P5(DEA,TEA)*DT-PE7(TEA,[illegible])*DT
S=[illegible]
D[illegible]=D[illegible]S
DTE=(DTE-TEA*[illegible]DDENS)/DEA
IF (ABS(DTE).GT.0.5*TEA)DTE=SIGN(0.5*TEA,DTE)
```

```
C [illegible] [illegible] TEMPERATURE
  DQ1=P[illegible](D[illegible],[illegible],TIA)*DT-P[illegible]3(DPA,D[illegible],T[illegible])*DT
 1-P16(DEA,TEA,TIA)*DT-P17(T[illegible]A,ADD(E))*DT
  DTI=(DQ1-[illegible]*[illegible])/[illegible]
  IF (ABS(DTI).GT.0.5*TIA)DTI=SIGN(0.5*TIA,DTI)
  TIA=TIA+DTI
  DAP=ADD(E)*DT
  AP=AP-DAP
C INCREMENT TIME
  II=II+1
  TIME=DT*FLOAT(II)
  GO TO 450
  CONTINUE
  STOP
  END
```